

Evolution to the Edge of Chaos in an Imitation Game

Motivated by the evolution of complex bird songs, an abstract imitation game is proposed to study the increase of dynamical complexity: Artificial "birds" display a "song" time series to each other, and those that imitate the other's song better win the game. With the introduction of population dynamics according to the score of the game and the mutation of parameters for the song dynamics, the dynamics are found to evolve towards the borderline between chaos and a periodic window, after punctuated equilibria. With topological chaos for complexity the importance of the edge of chaos is stressed.

1. INTRODUCTION

Increased complexity through evolution is believed to be visible in many biological systems, not only in the hierarchical organization in genotypes and phenotypes but also in animal behavior and communication. For example, a bird song increases its

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repertoire through evolution and developments.¹ In the bird song, two functions are believed to exist: one is defense of territory and the other is sexual attraction. A bird with a complex song (with many repertoires made from combinations of simple phrases) is stronger in defending its territory, as demonstrated by Krebs with the help of loud-speaker experiments.^{1,9}

There are reports of birds that try to imitate each other's song to defend their territory.¹ Inspired by these observations, we propose an imitation game of birds with their songs, used for territorial defense. A complex song may not be imitated easily and may be powerful in territorial defense. Although here we do not claim strongly the plausibility of this hypothesis, evolution in an imitation game is interesting as a novel evolution game¹² and in the more general context of the evolution of communication or mimicry.¹³ Here we consider an abstract game for imitation, where an artificial "bird" wins the game when it can imitate the other's song better than the other player or bird. If a song is simple, it may be imitated easily by others, and we may expect evolution towards a complex song.

The model we propose here has a sound basis in nonlinear dynamics. It adopts a nonlinear mapping as the generator of songs. Our birds thus can produce songs possessing the complexity of real numbers, which is one of the advantages of our model.

Another motivation of this chapter is the introduction of a simple model that realizes "evolution to the edge of chaos." Increased complexity at the edge of chaos has been discussed for cellular automata,¹⁰ for Boolean networks,⁸ and for coupled map lattices.⁶ It has been suggested that in a system with mutually interacting units, this edge of chaos has potential advantages in evolution.¹⁵ Although these studies are reasonable in their own contexts, there is no simple example that provides an evolution to the edge of chaos, in the sense of dynamical systems theory (note that chaos is defined only for dynamical systems with a continuous state and is not defined for a discrete-state system like cellular automata). Since we have adopted a nonlinear dynamical system as a song generator and an imitator, we can examine a song as it evolves towards the edge of chaos, which is a clear advantage of our model.

Indeed, we will find that the dynamics of the song time series evolves toward the edge of chaos through the imitation game. However, this edge state lies not at the onset of chaos, but at the edge between a window of a periodic cycle and chaos. The main difference is that chaotic orbits exist as a transient orbit in the windows. The importance of the existence of transient chaos will also be discussed.

2. MODELING

Our abstract model consists of the processes of song dynamics, imitation, game, and mutation.

A. SONG DYNAMICS

As a "song," we use a time series generated by a simple mapping $x_{n+1} = 1 - \alpha x_n^2$, the logistic map. (This map is equivalent to the logistic map of the form $z_{n+1} = b z_n(1 - z_n)$, by the simple transformation of variables $x_n = \frac{4}{b-2}(z_n + 0.5)$, $a = \frac{4(b-2)}{b}$. See any textbook on chaos for the bifurcation diagram of the logistic map.)

The attractor of the map shows a bifurcation sequence from a fixed point to cycles with periods 2, 4, 8, ..., and to chaos as parameter α is increased.^{2,1)} Parameter α is assumed to be different for each individual "bird." We choose this dynamics since it is investigated in detail as the simplest generic model for bifurcation and chaos.

B. IMITATION

Each bird player i chooses an initial condition so that, by its own dynamics $x_{n+1}(i) = f_i(x) = 1 - \alpha(i)x_n(i)^2$, the time series can imitate the song of the other player. Here we use the following imitation process for simplicity. For given transient time steps $T_{m,i}$, a player 1 modifies its dynamics with a feedback from the other player:

$$x_{n+1}(1) = f_1((1 - \epsilon)x_n(1) + \epsilon x_n(2)). \quad (1)$$

By this dynamics, the player 1 adjusts its value $x_n(1)$ by referring to the other player's value ($x_n(2)$). Here ϵ is a coupling parameter for imitation process. After repeating this imitation process for $T_{m,i}$ steps, the player 1 uses its own dynamics $x_{n+1}(1) = f_1(x_n(1))$. In other words, the above process is used as a choice of initial condition for the imitation of the other player's dynamics. The coupling parameter ϵ also varies by players. (However, the distribution of the parameter ϵ seems "irrelevant," judging from our simulations. We will discuss this parameter in later sections.)

C. GAME

We adopt a two-person game between "birds." After completing the imitation process, we measure the Euclidian distance $D(1, 2) = \sum_{m=1}^T |x_m(1) - x_m(2)|^2$ over certain time steps T . By changing the roles of players 1 and 2, we measure $D(2, 1)$. If $D(1, 2) < D(2, 1)$, player 1 imitates better than player 2 and thus is the winner of the game, and vice versa.

TOPOLOGY FOR THE PLAYERS. Here all "birds" are assumed to play the imitation game against all others with equal probability, although some simulations use a two-dimensional lattice with nearest-neighbor play and these are briefly discussed later.

After each game, the winner gets a point W , while a loser gets L ($W > L$). (Both get $(W + L)/2$ in the case of draw.) After iterating a large number of games, the population is updated proportionally to the score. This population update corresponds to reproduction, with survival of the fitter.

D. MUTATION

The parameters α ($0 < \alpha < 2$) and ϵ ($0 < \epsilon < 1$) can vary from individual to individual, and can be changed by mutation.⁵⁾ In the population update stage, mutational errors are introduced in to the parameters. The parameters α and ϵ are changed to $\alpha + \delta$ and $\epsilon + \delta'$, where variables δ and δ' are random numbers chosen from a suitable distribution (we use a homogeneous random distribution over $[-\mu, \mu]$ or Lorenzian distribution ($P(\delta) = 1/(\mu(1 + (\delta/\mu)^2))$). The former choice inhibits a large jump of parameters, and often the parameter values are trapped at intermediate values, whereas the latter choice is often useful, since it can provide a larger variety of species.

3. RESULTS OF SIMULATIONS

First of all, we start with a game without population dynamics. The score table of the game is plotted in Figure 1. We note that the outcome of the game can depend on the initial choice of $x_n(i)$ for the players. We have computed the score of player 1 with parameter $\alpha(1)$ against the player with parameter $\alpha(2)$, averaged from 100 initial values chosen randomly. In Figure 1 we plot the averaged score of the player with $\alpha(1)$ against the player with $\alpha(2)$.

Although the score table is rather complicated, we can see some strong bands of parameters against a wide range of parameters. Generally these parameters lie around the bifurcation points: bifurcation from a fixed point to period 2, from period 2 to period 4, etc. In addition to these period-doubling bifurcation points, the borderline between chaos and windows for stable cycles is stronger, in the high nonlinearity regime with topological chaos ($\alpha > 1.4011\dots$).

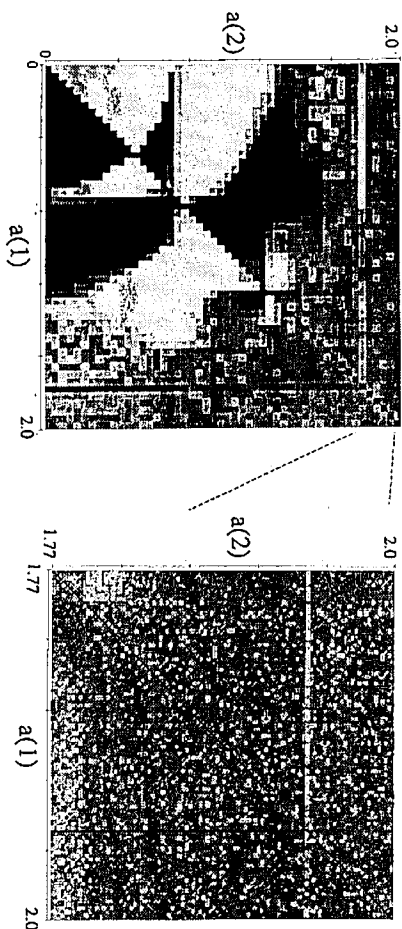


FIGURE 1 Score table of the initiation game. The score of the game between two players with parameters $a(1)$ and $a(2)$ is plotted with the help of a gray scale at the site corresponding to $(a(1), a(2))$. We have compiled the score of player 1 with parameter $a(1)$ against the player with parameter $a(2)$, averaged from 100 initial values chosen randomly. Blank corresponds to 100 wins of the player with $a(2)$, while black corresponds to 100 wins of the player with $a(1)$. The darkness of the site $(a(1), a(2))$ increases as the winning ratio of the player with $a(1)$ increases. The parameter a is changed by an increment of 0.04 in the left table and 0.003 in the right one. $T_{mi} = 30$ and $T = 16$.

With the inclusion of population dynamics, the score of each player can depend on the population at the moment; we do not have a fixed fitness landscape. To see a fitness tendency, we sample the scores for each parameter range over long time steps, taking a high mutation regime where many players of different parameters exist. The average score is shown in Figure 2 with the use of bins of 0.001. The ruggedness of the landscapes is clearly seen. For example, the peaks of hills are seen at 1.25, 1.38, 1.401, 1.625, 1.77, and 1.94. They correspond to the bifurcation points from periods 2 to 4, and periods 4 to 8, the accumulation point of period doubling bifurcations, and the edges of windows of periods 5, 3, and 4, respectively.

The importance of the (schematic) rugged landscape is discussed in various biological contexts. We have to note the following points in the present example. First of all, the fitness landscape is not given in advance, in contrast with many theoretical models.⁷ The rugged landscape is emergent through evolution. The landscape depends strongly on the population distribution of the moment. Second, the figure of the landscape is not schematic, in contrast with other landscapes, e.g., in spin glass models.¹⁴ Indeed, the horizontal axis of Figure 2 is just the parameter of the logistic map, while the axis for the spin glass-type rugged landscape is usually schematic.

Emergent Landscape: Score vs a

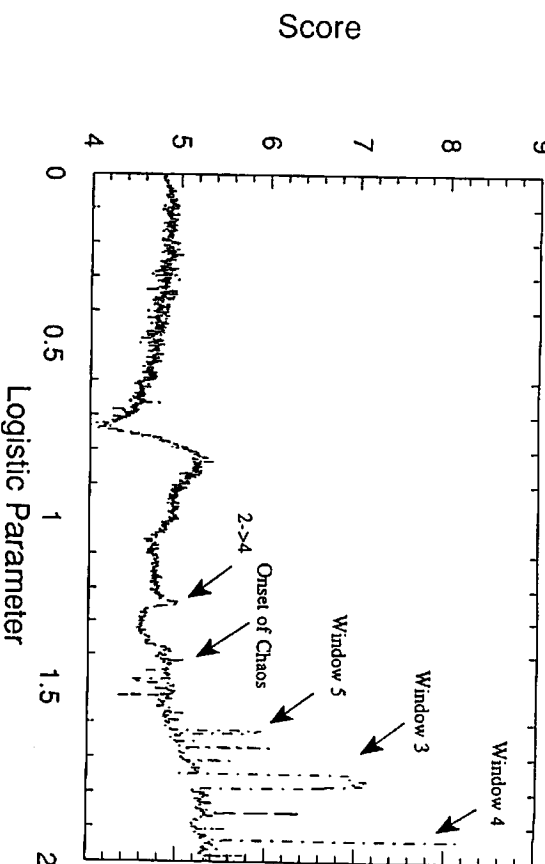


FIGURE 2 Emergent landscape: Average score for the players with parameters within $[a_i, a_i + \Delta]$ is plotted for $a_i = -1 + i \times \Delta$, with the bin size $\Delta = 0.001$. For Figures 2-4, we have adopted $W = 10$, $L = 1$, $T_{mi} = 255$, and $T = 32$. For mutational errors, random numbers δ are chosen from the Lorenzian distribution $P(\delta) = 1/\{\mu(1 + (\delta/\mu)^2)\}$, to avoid trapping at intermediate parameters. Simulation is carried out with the mutation rate $\mu = 0.1$, starting from the initial parameters $a = 0.6$ and $\epsilon = .1$. Sampled for time steps from 1000 to 1500, over all players (who always number 200).

Temporal evolution of the average of parameter a over all players is plotted in Figure 3. Plateaus corresponding to the hills in Figure 2 are observed successively, which provides an explicit example for punctuated equilibrium.³ At the temporal domain with a plateau, the parameter values of "birds" are concentrated on the plateau value. Each plateau corresponds to the period-doubling bifurcation or to the edge of periodic windows among chaotic states. For example, the plateaus in Figure 2 are the superstable point of chaotic orbit ($a = 1$, where the derivative $f'(x_1)f'(x_2)$ changes its sign),^{11,2} the bifurcation point to period 4, and then to period 8, and the edge of the period 3 window, and finally the edge of the period 4 window. As the mutation rate is decreased, plateaus have longer time intervals, and many steps corresponding to finer windows (and the accumulation point of period doublings) are observed.

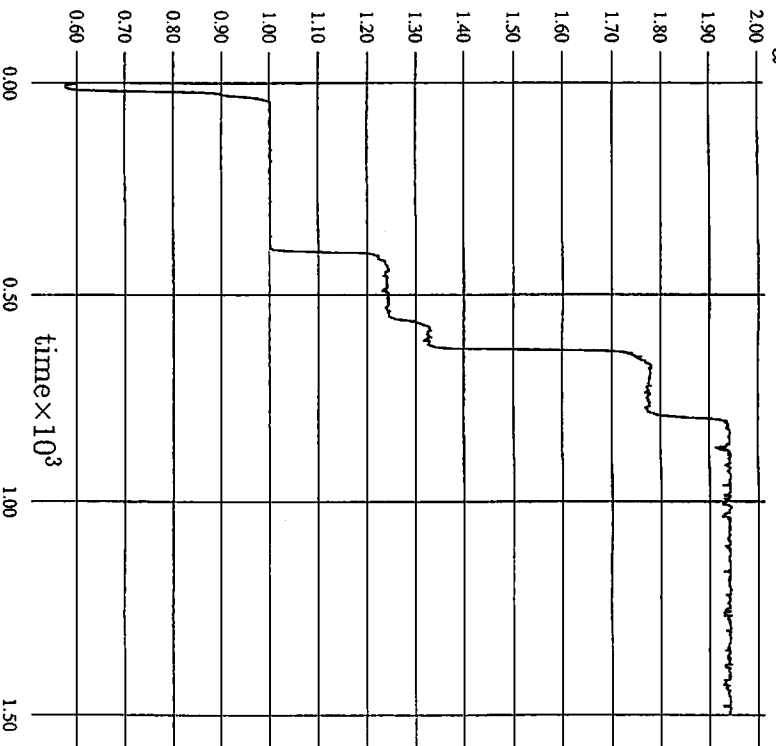


FIGURE 3 Temporal change of the average parameter α : the simulation is carried out with the mutation rate $\mu = 0.0005$, starting from the initial parameter $\alpha = 0.6$ and $\epsilon = .1$. The average of the parameters α over all players are plotted with time. The total population is fixed at $N = 200$.

Evolution leads our system to the edge of chaos, i.e., the borderline between chaos and a window. To see clearly "the edge of chaos," we have measured the Lyapunov exponent λ . The Lyapunov exponent is a characteristic of asymptotic orbital instability, given by the small growth rate between two close orbits. The exponent is positive for a chaotic orbit, whose magnitude characterizes the strength of chaos, while it is negative for a stable periodic orbit. Thus a border between chaos and a periodic state corresponds to $\lambda = 0$. In Figure 4 we have plotted the birds' score as a function of the Lyapunov exponent λ , sampled for 200 time steps at the final stage of evolution. As shown, the score has a broad peak around $\lambda = 0$.

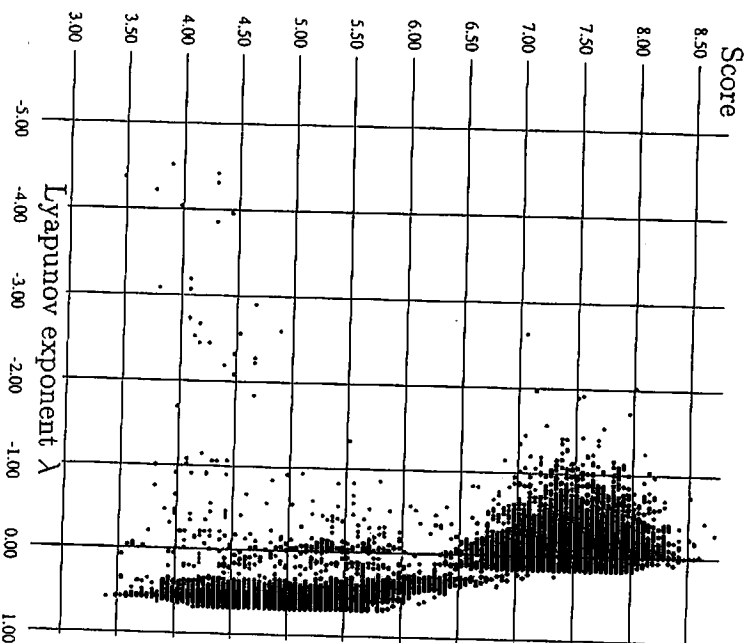


FIGURE 4 Lyapunov exponent versus score: the simulation is carried out with the mutation rate $\mu = 0.001$, starting from the initial parameters $\alpha = 0.9$ and $\epsilon = .1$. Lyapunov exponents of the song dynamics vs. the score against all players at the moment are plotted. Sampled for time steps from 600 to 800 over all players (who always number 200), where the parameter value α is concentrated around 1.94.

Thus we have observed the evolution of a song to increase its complexity in our imitation game. The dynamics of the song time series evolves towards the edge of chaos. Although there is no "strongest" parameter in a two-player game, the borderlines between periodic windows and chaos are selected in the course of evolution. The song at the edge ($\alpha \approx 1.94$) of the period 4 window is strong and robust.

The robustness of the parameter depends on the mutation rate. In the landscape (Figure 2), another important quantifier besides its height is the width of a hill. For example, the hill around $\alpha \approx 1.94$ is higher than the hill around $\alpha \approx 1.77$, but

the latter hill is much wider. A wider hill is more robust against the variation of the parameter and is more advantageous in a higher mutation-rate regime. Indeed, in a simulation with a higher mutation rate, the population is concentrated around $\alpha \approx 1.77$. The width of a hill is governed by the width of a window: thus the edge of a wider window is robust at a higher mutation rate.

We have made a large number of simulations, by changing parameters and initial conditions. Our conclusions are invariant against these changes of parameters, unless the mutation rate is too small for us to observe the successive changes within our simulation time steps (usually less than 10^5 steps).

4. MODIFICATIONS AND EXTENSIONS

There can be many generalizations to our model. Here we will briefly discuss some of these, although the results are still rather preliminary.

1. Change of the topology: Instead of the game of all-to-all, it may be interesting to study a game on a two-dimensional lattice with nearest-neighbor interaction. Most of our results in Section 3 are reproduced, although spatial differentiation and clustering are found in the lattice version for a suitable mutation rate. The coexistence, for example, of the species near the period 3 and period 4 windows is seen in this case. Needless to say, "fluctuation" is much bigger in this model than that presented in Section 3. Consequentially, some plateaus observed previously are lost depending on the value of mutational errors.
2. Symbolization of a song: Instead of judging the imitation by the Euclidian distance, we could check it after symbolizing the dynamics to few digits (here we choose 2). It is well known that the time series of the logistic map can be traced by the binary sequence (left, i.e., $x < 0$; right, i.e., $x > 0$). We check if the sign of x_m is identical between players 1 and 2, at each time step, and count the number of time steps where the symbols are not matched. In other words, we use the following function $I(1, 2)$ instead of $D(1, 2)$: $I(1, 2) = \sum_{m=1}^T \text{sign}(x_m(1), x_m(2))$ with the notation $\text{sign}(a, b) = 1$ if $ab < 0$ and 0 otherwise. Adopting this criterion, we have performed simulations of our game on a two-dimensional lattice. We have found that the logistic parameter α of winners often lie at a point where the binary sequence changes its pattern, the right-left sequence. Here, the temporal behavior of evolution is more complicated than that in Section 3. A punctuated equilibrium around $\alpha = 1.77$ can be established for some intervals; however, this does not continue over a long time scale, and switches occur to different transient states. Often species with distant parameters can coexist. Generally speaking, complexity and diversity are enhanced in the present case with symbolization. This result may give some hint to the origin of complexity and diversity in communication and language.

3. Dual dynamics for song and imitation: What happens if birds assume different parameters in singing and in imitating? This would be a reasonable question, since there is no primary reason that the two processes are governed by identical dynamics. From some simulations, the parameter for the singing process again shows punctuated equilibria among "window" values, before settling down to the border between chaos and the period 4 window value ($\alpha \approx 1.94$). The parameter for the imitation process, on the other hand, is often settled around $\alpha = 0.75$, the bifurcation point from a fixed point to period 2.

If we adopt the symbolization dynamics as in (2), the dynamics is again more complicated. Again, the parameter for the imitation process shows punctuated equilibria among "window" values. However, the parameter does not settle to a window value (e.g., at $\alpha \approx 1.94$), but wanders upwards and downwards forever. On the other hand, the parameter for singing often increases up to the value for fully developed chaos, although there is a small amplitude variation, in synchronization with the change of the parameter for imitation.

4. Changes in the choice of mapping: This would be the most immediate idea for most of us. It would be far more interesting to include evolution, not only within the logistic maps with a variable parameter, but also towards different types of maps.

5. DISCUSSIONS

In this chapter we have presented a "minimal" model showing evolution to the edge of chaos and punctuated equilibrium. Indeed, it gives the first explicit example for this type of evolution in the sense of dynamical systems, as is demonstrated by evolution to a state with a Lyapunov exponent close to zero. In addition, we note that our edge state lies between a window and chaos. A logistic map has topological chaos at the window and can show chaotic transients before the dynamics settles down to a stable cycle. The existence of transient chaos is useful in the imitation of transient dynamics. In the window regime, the logistic map includes a variety of different orbits, some of which are close to a periodic orbit, while others are chaotic. A window at a higher nonlinearity regime includes a variety of unstable cycles, as coded by Sharkovskii's ordering,^{2,11} and can provide a larger variety of dynamics as transients. This is the reason why the edge of the window is strong in our imitation game. The above speculation suggests the importance of transient chaos, besides the edge of it, for the adaptation to a wide range of external dynamics.

Our imitation process is based on the synchronization of a player's dynamics to the other's song. This synchronization process may not be physiologically unrealistic. Indeed, there have been recent extensive studies on the entrainment of oscillation. So far the studies on synchronization is mostly focused on the visual

cortex,⁴ but there is no reason to suspect the importance of synchronization in auditory systems.

Evolution of complexity to escape from imitation may play a key role in many fields. Our imitation game provides one route to the evolution to complexity. We may expect that this route can be seen in some examples of biological evolution, besides our original motivation of a bird song. One is the evolution of a communication code only within a given group: "secret code." When there are two groups, it is often important to send some messages only within the same group, so that they are not encoded by the other. Another example is the Batesian mimicry.¹² Here again, we have two groups. One of the groups can survive better by imitating the pattern of the other's group, while the other group's advantage in survival is lost if it is not distinguished well from the other. Complexity in the patterns of butterflies in the mimicry relationship, for example, may be increased through the imitation game process. Our simulation gives the first conceptual model for the evolution to complexity by imitation, while modeling with two distinct species with different roles will be necessary in the future.

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