ELSEVIER

Physica D 75 (1994) 328–342

PHYSICA D

Imitation games

Junji Suzuki[a], Kunihiko Kaneko[b]

[a] Institute of Physics, College of Arts and Sciences, University of Tokyo, Komaba 3-8-1, Meguro-ku, Tokyo 153, Japan
[b] Department of Pure and Applied Sciences, College of Arts and Sciences, University of Tokyo, Komaba 3-8-1, Meguro-ku, Tokyo 153, Japan

Abstract

Mutual-imitation games among artificial birds are studied. By employing a variety of mappings and game rules, the evolution to the edge between chaos and windows is confirmed. Some other general features are observed, including punctuated equilibria, and successive alternations of dominant species with temporal complexity. It is also shown that diversity of species is drastically enhanced if the songs are represented by discrete symbols.

1. Introduction

The evolution of species in mutually interacting systems has been a problem of interests in many branches of science. Is there a trend to increase the complexity? Is there a characterization for an evolutionarily favorable state? The determination of such a state is generally a complicated question involving a certain balance among several factors, such as costs to adopt a strategy and behavior, and gains of adopting it.

The recent "dogma" for the characterization may be the "edge of chaos" scenario. This concept has been emphasized in many contexts such as cellular automata, Boolean networks or CML [1–4]. To the authors' knowledge, however, there was no clear example presenting the evidence for evolution to the edge of chaos in the exact sense of dynamical systems theory.

In a recent communication, we have proposed a minimal model for the evolution to the edge of chaos, based on the dynamical systems theory [5]. The model was motivated by the observed complexity of bird songs: It is known that a bird with a complex song (with a large repertoire based on combinations of simple phrases) is stronger in defending its territory, as is observed by Krebs with the help of loudspeaker experiments [6,7].

So far the reason why a complex song evolves is unknown, although there is the hypothesis that a complex song may give the impression of a crowd, thus being efficient for defense [7]. This hypothesis, however, has no experimental evidences.

On the other hand, there are some reports of observations that birds try to imitate each other's songs for the defense of territory [6]. Combining the above two observations, the following hypothesis has been put forward for the explanation of the complexity of bird songs: Birds join a battle for defense of territory through mimicking each other's songs. In other words, if a bird A can imitate bird B's songs better, then A can

0167-2789/94/$07.00
SSDI 0167-2789(94)00065-X

intrude B's territory and share food. A is advantageous in the battle for survival.

It may be expected that a complex song is difficult to imitate. Hence we may expect the evolution to a complex song. Despite a lack of direct experimental support for this scenario, the concept of mutual-imitation game itself deserves much attention, as a novel problem of game theory and as a general framework for the evolution to complex behavior.

In the abstract imitation game, we assume that songs of an artificial bird (or player) are generated by a nonlinear map. Birds play imitation games with each other. We assume that the winner of the game is fitter for the survival, which is taken into account by the population dynamics as a growth rate or by a simple replacement of losers by winners. Mutational changes of parameters for the song dynamics are introduced in the course of replication (see Section 2 for modeling details).

We have to emphasize two advantages of our modeling here.

- Since our artificial birds generate their songs by nonlinear mapping, songs can have a complexity generated with real numbers. In most game theory, strategies are chosen from discrete sets, while the strategies (songs) here are chosen from continuous sets.
- Since our parameters have a clear meaning in the nonlinear mapping, the meaning of the edge of chaos is quite clear in the exact sense of dynamical systems theory.

Since all players are labeled by continuous parameters, no clear speciation is given in advance. Judging from the result of simulations, however, it seems justifiable to divide players into several species.

With high mutation rates, the coexistence of several species has been observed, while, by lowering the mutation rates, we have explicitly shown the evolution to the edge of chaos, in the exact sense of nonlinear dynamics. In addition, the evolution occurs towards the borderline between a window and chaos, not to the first onset of chaos. This observation leads to the hypothesis of the evolution to the border between observable and invisible chaos. Some other features emerge through the evolution, such as punctuated equilibria and abrupt, complex alternations of dominant species.

In the previous report [5], we have adopted

- the logistic map as a song generator,
- a Euclidean criterion for imitation,
- an "infinite dimensional" lattice (all-to-all game),

(see the meanings of these terms in Section 2).

Here we report studies on several variants of the model to test the validity of the results and the argument in the previous communication [5]. For example, we put birds on a 2D square lattice, allowing only fights among the players on the nearest neighbor sites. In most cases, the results in the previous paper [5] are reproduced, implying the universality of the concept of the "edge of chaos", and the "edge of windows". Furthermore, we find several exotic phenomena such as the diversity (or coexistence) of species assisted by discrete symbolization of birds' songs.

This paper is organized as follows. In the next section, we present the details of our modeling. In Section 3, we survey the results of the model with the "Euclidean criterion", parts of which were also reported previously [5]. These results provide basic notions and a framework for later sections. Characteristic features of the imitation games are shown there. We will give some intuitive reasons why the system evolves to the edge between chaos and a window. Moreover, it will be shown that our "edge of chaos" scenario is stable against the choices of the topology of players. In Section 4, we present some results, obtained with the use of digital symbol representation of songs, "LR criterion". Such symbol representation of dynamics is shown to lead to an increase of fluctuations, resulting in the diversity of species. In Section 5, we resume the problem of the priority of the edge between chaos and a window by changing the birds' song generator (to the tent or the circle map). In particular, we will confirm its priority by simulating a system with the coexistence of two distinct types of song mappings with and without windows (i.e., the logistic and tent maps). In Section 6, we will discuss the complex dynamics of the imitation games by allowing for different dynamics for the "song generation" and imitation processes. A summary and discussions are given in Section 7.

2. Modeling

Our model consists of three stages: the song generation process, a 2-persons' imitation game and reproduction. We will discuss these three procedures separately.

2.1. Song generation (song dynamics)

As a "song", we use a time series generated by a nonlinear map. In the previous report [5], the logistic map

$$x_{n+1} = 1 - ax_n^2 \tag{1}$$

was adopted for this purpose, since it is one of the most thoroughly investigated maps and a standard model in nonlinear physics. A bird, say, the ith bird, possesses its own parameter $a(i)$. We regard the time series $(x_0(i),\ x_1(i), \ldots)$ generated by the map, as the ith bird's song.

In simulations here, the logistics map is again chosen, while other maps for a bird's song generator are also used to test the universality of our results. Here simulations with the use of the tent map or the circle map will be given in Section 5.

2.2. 2-persons imitation game

Three factors are involved in defining the 2-persons' game: topology of players, imitation and game processes (or criterion for winners). Here we give a detailed explanation of each process separately.

2.2.1. Topology of players

In the previous report [5], players are assumed to fight against all other players. In the sense of statistical physics, players live on an infinite dimensional lattice. Spatial information is not included in this modeling. Here we also investigate cases where the players live on a two-dimensional square lattice with periodic boundary conditions. For most simulations, we adopt a lattice of 30×30.

2.2.2. Imitation process: choice of an initial value for song imitation

Each bird player has to adjust the initial condition for its song so that the time series by its own dynamics can imitate the other player's song better. Here we use the following imitation process for simplicity. Each player generates its song starting from a random initial value (within $[-1,1]$) and first repeats its own dynamics T_0 times to eliminate transients. Then, for a given transient time T_{imi}, player 1 (mimicker) modifies its dynamics with a feedback from player 2 (singer):

$$x_{n+1}(1) = f_1[(1-\epsilon)x_n(1) + \epsilon x_n(2)]\,. \tag{2}$$

Here ϵ is a coupling parameter for the imitation process. After repeating this imitation process for T_{imi} iterations, player 1 uses its own dynamics $x_{n+1}(1) = f_1[x_n(1)]$. In other words, the above process is used as a kind of initial condition for the imitation of the other player's dynamics. The coupling parameter ϵ also varies by players. However, the distribution of this parameter seems "irrelevant", as far as we judge from our numerical results. Thus we will skip the discussion on this parameter.

2.2.3. Game

After player 1 completes the above imitation process, the two players are decoupled and generate songs by their own dynamics. Then we calculate a quantity $D(1,2)$ measuring the distance between the imitated time series $\{x_n(1)\}$ and the singing one $\{x_n(2)\}$. By changing the role of two players, $D(2,1)$ is also measured. If $D(1,2)$ is smaller than $D(2,1)$, player 1 imitates the other's song better and wins this 2-persons' game (and vice versa). The definition of the distance $D(i,j)$ gives a criterion for the winner and may be crucial in the game. Here we adopt either of the following two criterions:

(1) *Euclidean distance.* As the measure, we choose $D(1,2) = \Sigma_{m=1}^{T} |x_m(1) - x_m(2)|^2$ over a certain time T. This choice is just a normal Euclidean distance. (Of course, the distance should be cordial for the circle map, since x is on a circle).

(2) *LR-code.* Instead of using real numbers $\{x_n(i)\}$ for the criterion, it is often useful to represent them by some discrete set of symbols. The time series of the logistic map, for example, can be traced by two symbols, represented by R ($x > 0$) and L ($x < 0$) [10]. By using such symbolization, the distance between two trajectories is measured by the number of unmatched symbols over some iterations. In the above symbolization for the logistic map, the distance is thus defined by

$$D(1,2) = \sum_{m=1}^{T} \mathrm{IS}(x_m(1), x_m(2)), \qquad (3)$$

where $\mathrm{IS}(a,b) = 1$ if $ab < 0$ and 0 otherwise.

2.3 Reproduction

The fitness of the players is given by the results of the games. Offsprings are produced according to the results of fights. We adopt either of the following two for this process:

(a) *Replacement.* This might be the simplest way; the parameters of losers are replaced by those of winners at every game. We adopt this replacement rule for players on a 2D lattice.

(b) *Score.* After each game, the winner gets W points, while a loser gets L points ($W > L$) (Both get $\frac{1}{2}(W + L)$ in the case of a draw.) After a large number of games, the population distribution is updated proportionally to the score of the players, with the further restriction that the total population be constant.

At both of these reproducing stages, we include mutational errors in the parameters; the parameters a and ϵ are changed to $a + \delta$, $\epsilon + \delta'$, respectively, where variables δ and δ' are random numbers chosen from a suitable distribution (we use a homogeneous random distribution over $[-\mu, \mu]$, or a Lorentzian distribution ($P(\delta) = 1/\{\mu[1 + (\delta/\mu)^2]\}$). The latter choice is often useful, since the former choice inhibits a large jump of parameters and often the parameter values are trapped at intermediate values, while the latter can provide a wider range of "species".

These three processes define our model. We will sometimes specify a model by its sets of rules such as [2D lattice, logistic, LR, score, 5×10^{-4}], which means that players are on a 2D lattice, and generate songs by the logistic map, that the distance between two songs is measured by the LR symbolic sequence, and that the population is updated proportionally to player's score with the mutation rate $\mu = 5 \times 10^{-4}$.

Throughout this paper one time step is defined such that every player gets one turn, where one turn is a game with one of the neighbors in the lattice case and a game with all in the all-to-all case. Thus there are in total N games for N players for the lattice case, and $\frac{1}{2}N(N-1)$ games for the all-to-all case.

3. Basic results: simulations with the Euclidean criterion

Let us start our presentation by giving numerical results with the Euclidean criterion for distance.

First, we present the results of a game with high mutation rates. The model is defined by [2D lattice, logistic, Euclidean, replacement, 0.001]. In Fig. 1, we present the distribution of the logistic parameter a after 10 time steps. It clearly shows the coexistence of two species; one is

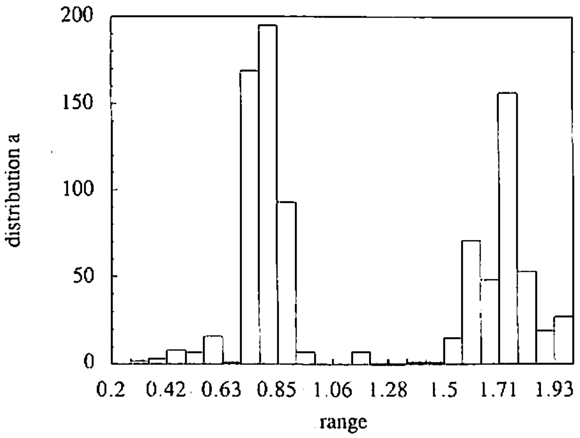

Fig. 1. Snapshot distribution of the logistic parameter of the players after 10 time steps. The mutation parameter μ is 0.001, and the lattice size is 30×30. Initially a set of parameters $\{a, \epsilon\}$ for a bird is assigned randomly from [0.6,1.1] and [0,0.5], respectively. Two peaks corresponding to special points for the logistic map are observed (see text). Unless otherwise mentioned, we set $T_0 = T_{\text{imi}} = T = 30$ throughout the present paper.

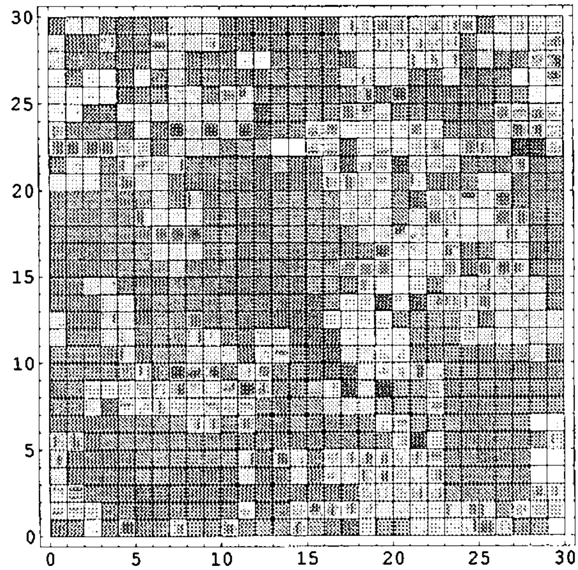

Fig. 2. Snapshot pattern of the configuration of distribution of the logistic parameter, corresponding to Fig. 1. Here a bright (dark) pixel corresponds to a player possessing $a \sim 1.75$ (0.75), respectively.

around $a = 0.75$ and the other is around $a = 1.75$. The former corresponds to a bifurcation point (period-1 $\rightarrow$ period-2) while the latter corresponds to the edge between the period-3 window and chaos. The snapshot configuration is depicted in Fig. 2, where dark dots mean players with $a \sim 0.75$ while bright dots give those with $a \sim 1.75$. This example already shows the characteristic phenomenon that advantageous states are near bifurcation points (in other words, the "edge of something").

To investigate the fitness of the players we switch to a model with all-to-all games [infinite, logistic, Euclidean, score], as presented in the previous communication, since it serves as a prototype for our models.

The average score of players is plotted as a function of the bifurcation parameter a (Fig. 3). By taking a high mutation rate, we allow for the existence of players with a wide range of parameters a. We can see many peaks which correspond to the the bifurcation points from period-2 to period-4, and from period-4 to period-8, and to the edge of the period-3 window, period-5 window, period-4 window, and so forth. This score landscape is rugged as is often discussed in spin-glass type models in biology [8,9]. Here this landscape is not implemented as a model itself (like the energy function in spin-glass type models [9]), but is emergent through the evolution. Indeed this landscape depends on the population distribution at the moment.

To see the advantage of the "edge of chaos" in the score, we have plotted the score as a function of the Lyapunov exponent λ (Fig. 4). As is shown there, the score has a clear peak around $\lambda = 0$. Indeed this peak stems from the edge of the period-3 window and chaos ($a \approx 1.75$), and the edge of the period-4 window and chaos ($a \approx 1.94$).

A high mutation rate, however, leads to large fluctuations, which make it difficult to identify each "species" precisely. Let us therefore concentrate on low mutation rate cases for the rest of the present paper.

In Fig. 5, the temporal evolution of the average of the parameter a over all players is plotted. Plateaus are observed successively, providing an

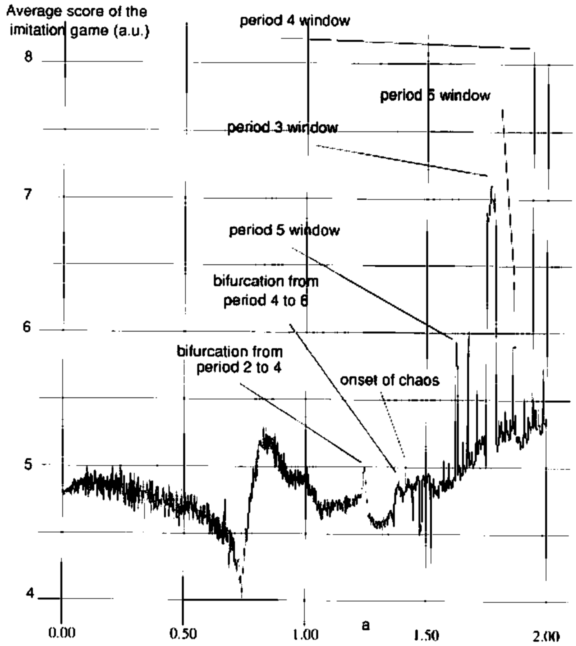

Fig. 3. Emergent landscape: Average score for the players with parameters within $[a_i, a_i + \Delta]$ is plotted for $a_i = -1 + i\Delta$, with the bin size $\Delta = 0.001$. We have adopted $W = 10$, $L = 1$, $T_{imi} = 255$ and $T = 32$. The simulation is carried out with the rule [infinite, logistic, Euclidean, score, 0.1], starting from the initial parameter $a = 0.6$ and $\epsilon = 0.1$. Sampled for time steps from 1000 to 1500, over all players (whose number is fixed at 200). (Adapted from [5].)

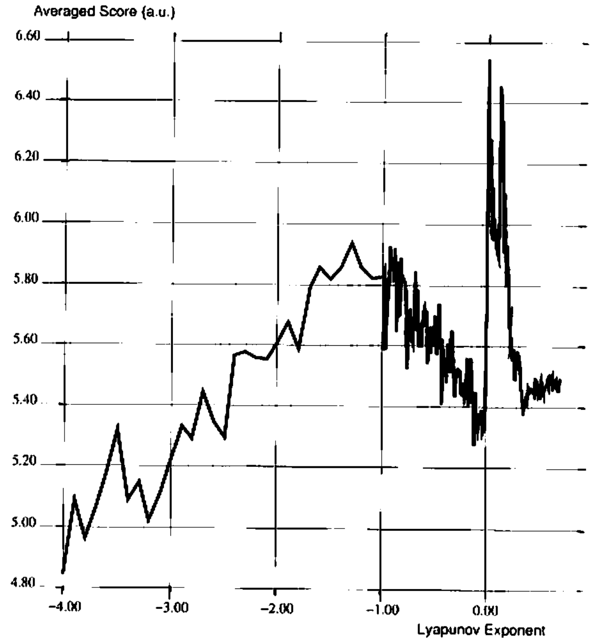

Fig. 4. Average score of the game versus Lyapunov exponents. The simulation is carried out with the rule [infinite, logistic, Euclidean, score, 0.02], $T_{imi} = 255$ and $T = 32$. We have adopted $W = 10$, $L = 1$, starting from the initial parameters $a = 0.9$ and $\epsilon = 0.1$. Average scores are obtained from the histogram of Lyapunov exponents, for which we use a bin size of 0.01 for $-1 < \lambda < 1$, while it is set at 0.1 for $\lambda < -1$ (since the sample there is rather sparse). Sampled over time steps from 500 to 750 over all players (whose number is fixed at 200).

explicit example for punctuated equilibrium [11]. In a temporal domain with a plateau, the deviation from the average value is typically very small. Each plateau corresponds to a bifurcation point or an edge of chaos as mentioned for the score landscape. Finally, the birds reach the parameter for the edge of the period-4 window ($a \approx 1.94$) and stay there [1].

Thus we have seen the priority of the "edge of chaos", and the evolution to it. To be more precise, we have to note that our edge state lies between a window and chaos. In windows, the logistic map can provide chaotic transients before the dynamics settles down to a stable cycle. Thus the existence of transient chaos should be useful to imitate a dynamics of a different nature. A window at a higher nonlinearity regime includes a variety of unstable cycles, as coded by Sharkovskii's ordering [10]. Therefore it can provide a larger variety of dynamics, as transients. This might be the reason why the edge of windows is strong in our imitation game. The above speculation suggests the importance of transient chaos, besides the edge of chaos, for the adaptation to a wide range of external dynamics.

[1] The averaged score plotted as a function of the Lyapunov exponent λ has a broad peak around $\lambda = 0$, which extends to the region $\lambda < 0$ rather deeply. The score shows a sharp drop at the side of $\lambda > 0$, on the other hand. With the increase of T, the peak slightly shifts in the positive direction while the drop for $\lambda < 0$ gets sharper.

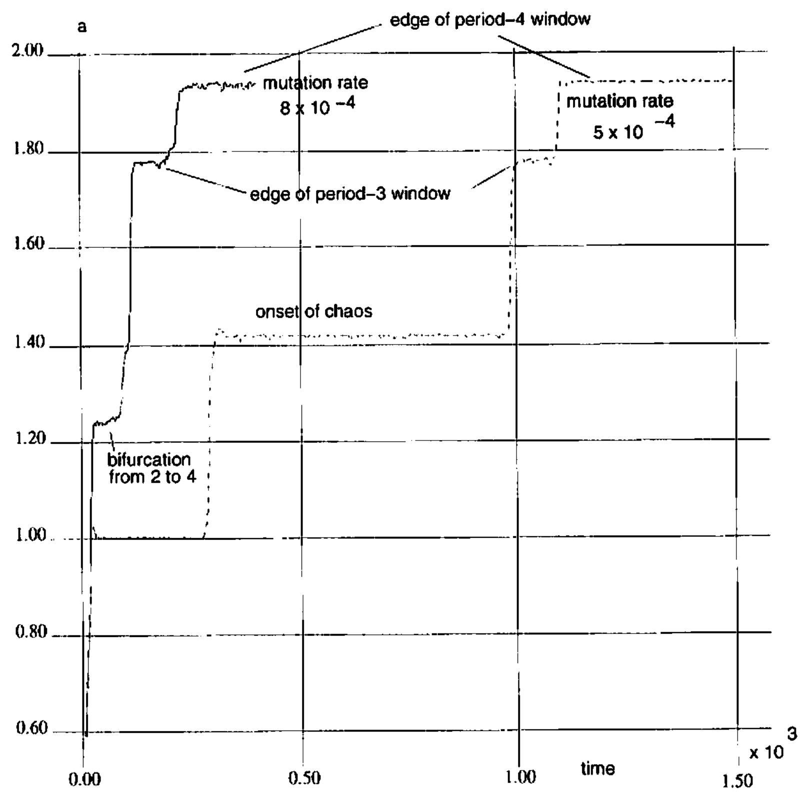

Fig. 5. Two examples of the temporal evolution of the average of the parameter a: The simulation is carried out with the rule [infinite, logistic, Euclidean, score], and with mutation rates 5×10^{-4} and 8×10^{-4}, $T_{imi} = 200$ and $T = 50$, starting from the initial parameters $a = 0.6$ and $\epsilon = 0.1$. Averages of the parameters a over all players are plotted versus the time. The total population is fixed at $N = 200$.

Let us incorporate again the lattice structure, by taking the model [2D lattice, logistic, Euclidean, score, 8.7×10^{-5}]. The temporal evolution of the average logistic parameter value a is plotted in Fig. 6. Again we see the evolution to the edge of chaos (windows) after punctuated equilibria. Thus the previous model with an all-to-all game can be regarded as a good mean field theory for our problem. The lattice structure enhances the fluctuations as is seen by comparing the flatness of the plateaus in Figs. 5 and 6 or noticing the lack of some plateaus here. To get some spatial information, we measure the distribution of the maximum connected cluster sizes of a species, since admissible parameter regions are separated enough to be regarded as distinctive species. We have plotted the temporal changes of the maximal cluster sizes around the time steps for abrupt changes of the mean logistic parameter (Fig. 7). Successive transitions of dominant species are clearly visible.

An interesting question here is if the lattice structure helps the coexistence of species or not. In Fig. 7, both species with $a \approx 1.75$ and $a \approx 1.86$ increase their populations together, which seems to suggest a kind of symbiosis. This example, however, is not so decisive to claim that spatial differentiation supports the coexistence of species. Further studies are necessary.

The examples in the present section show that the "edge of chaos" concept is valid irrespective of the topology of players. We stress again the

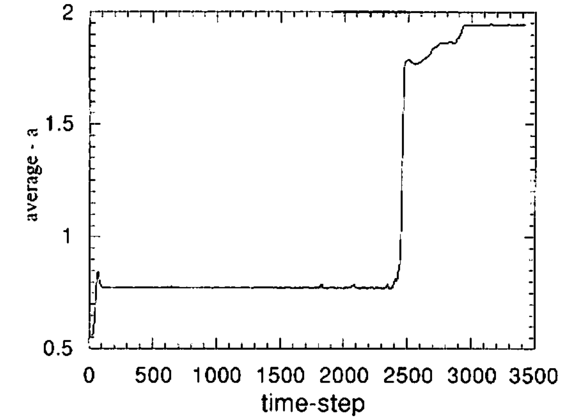

Fig. 6. Temporal evolution of the average logistic parameter a for [2D lattice logistic, Euclidean, replacement, $\mu = 8.7 \times 10^{-5}$] (as mentioned in the text, one time step = 900 2-persons' game). Initial values for a are chosen randomly from [0.6,1.1.]. Punctuated equilibria at $a \sim 0.77$ and ~ 1.94 can clearly be seen. The plateau at 1.94 continues after 3500 steps, as far as we have observed.

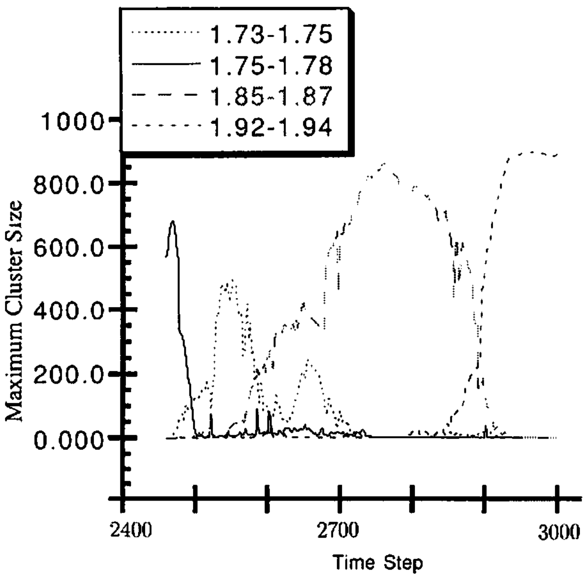

Fig. 7. Temporal evolution of four connected clusters in the vicinity of the change of the average logistic parameter in Fig. 6. Around the time steps [2700,2800], two clusters (~ 1.74 and (~ 1.86) seem to grow simultaneously.

significant role of the windows, which will be further addressed in Section 5.

4. Symbolization induced complexity and diversity; LR criterion

Next we study the case with the use of the distance in terms of discrete symbolic codes. The motivation for this choice is the examination of the effect of symbolization in communication codes: Does representation by a discrete set of symbols lead to an increase or decrease of diversity and complexity?

The temporary evolution of the average logistic parameter is given in Fig. 8. In this case, advantageous players have such parameters that the symbolic sequence changes its pattern there, e.g., $a = 1, 1.3, \ldots$. It is not difficult to see that the dominant species lie again at window values for long intervals. Clearly, this example shows the stability of the "edge between chaos and window" scenario against the choice of criterions in the 2-persons' game.

Judging from the amplitudes in Figs. 7 and 8, we conclude that the fluctuations here are larger

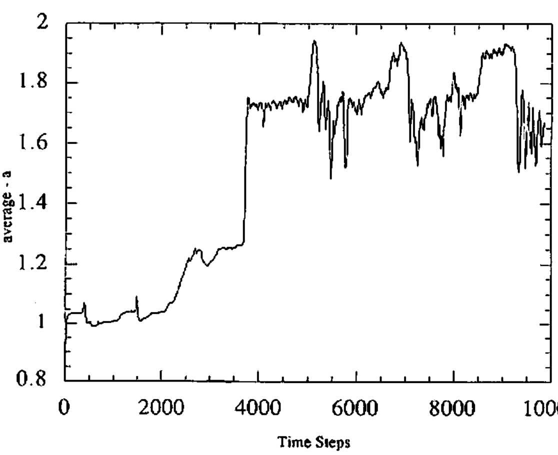

Fig. 8 . Temporal evolution of the average logistic parameter a for [2D lattice, logistic, LR, replacement, $\mu = 1.25 \times 10^{-4}$]. Alternations between states at $a \sim 1.77$ and ~ 1.94 are seen.

than in the Euclidean criterion case. In the lattice version, the average values of a go up and down temporally with some transient "disordered" intervals. No "final" parameter exists here, in contrast with the case of the Euclidean criterion, where $a \approx 1.94$ is robust and is reached from any initial distributions of parameters. The parameters switch (almost) forever. After staying at the edge between some window and chaos, the parameter switches to another edge or to a bifurcation point. In the example of Fig. 8, successive transitions between $a \approx 1.75$ (period-3) and $a \approx 1.94$ (period-4) are observed. Such complex alternations between "ordered states" are typically seen in the chaotic itinerancy [12], although the present switching process may be of a stochastic (nondeterministic) nature.

Moreover, coexistence of different species (parameters) in the LR case is strongly enhanced, compared with the Euclidean case with the same mutation rate. See Fig. 9 for snapshots of the distribution of players with respect to distinct sets of the logistic parameter a. The diversity is enhanced for the symbolic criterion.

The increase of diversity is also seen in simulations with the all-to-all game. Roughly speaking, players with $a \approx 1.75$ (period-3 window) are rather robust and occupy a large ratio of the population. (As the fluctuations increase, a window with a larger interval (such as the period-3 one) is more robust, which is also true of a model with a high mutation rate with the Euclidean criterion [5].) Still, we have seen coexistence of other parameters such as $a \approx 1.94$ besides the above group. When the average is plotted versus time, it fluctuates around $a \approx 1.75 \sim 1.78$, while the variance of the fluctuations remains large even in a very small mutation rate regime (say 10^{-4}).

Summing up the present section, we have found the symbolization enhances complexity of dynamics and diversity of species. We have found coexistence of two groups, and successive switches of dominant species forever.

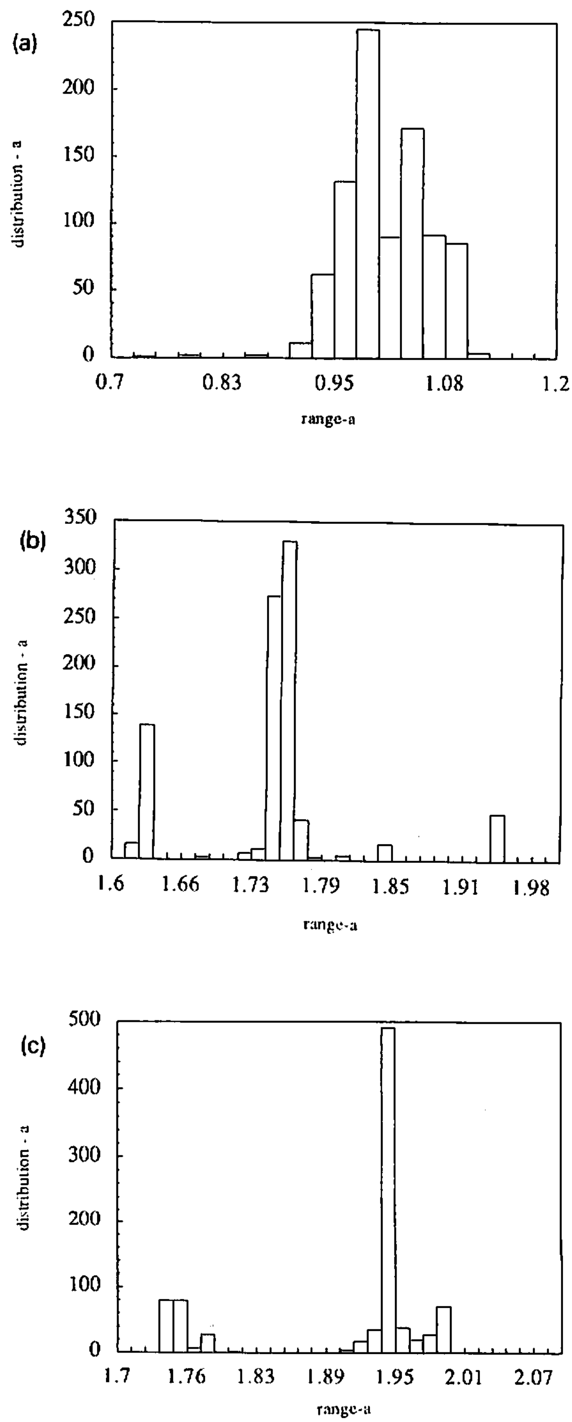

Fig. 9. Successive snapshot distribution of the logistic parameter at time steps 1000, 5000, 7000. The parameters of the model are taken identical to those in Fig. 8.

What is the origin of this diversity and complexity? One possible guess is the following. By the binary symbolization, players with a certain range of the logistic parameter come to possess an identical temporal pattern in singing, whereas fine structures within the parameter interval are distinguished in the Euclidean (or analogue) measure. Thus a "strong" pattern of time series which was observed only in some very narrow widow regime may be stabilized and exists in a wider parameter regime. The species, for example at $a \sim 1.63$, can have a larger chance for survival.

By the above mechanism of symbolization, effective strength of players is further averaged out. The difference between two songs in the game is much weaker than the Euclidean case [2]. Relationships between two players with different edges are more subtle, and no robust parameter exists. The switching dynamics and the coexistence may reflect this averaging effect by the symbolization.

If this scenario of diversity is universal, it suggests the importance of discretization to symbols, for the diversity. Why is the human language so diverse? Is the diversity caused by our ability to represent analogue vocal signals by a discrete (digital) set of symbols?

5. The priority of the edge between a window and chaos

Here we examine the scenario in Section 3, by choosing a few different mappings for the song dynamics. In this section, we will use the tent map and the circle map as the song generator. The results support our hypothesis on the priority of the windows, as discussed in Section 3.

5.1. Tent map

We choose the following tent map parameterized by one parameter,

$$x_{n+1} = a(\tfrac{1}{2} - |\tfrac{1}{2} - x_n|)\,, \tag{4}$$

as the song dynamics. Thus each bird possesses two parameters $a(i)$, $\epsilon(i)$ as in the case of the game with the logistic map. In the tent map, there is no window structure. With the increase of a, chaos appears at $a = 1$, which is the only edge of chaos. For $a > 1$, no bifurcation structure exists.

Numerical results of the evolution of the imitation game show that the average value of the parameter a evolves to 1, the onset of chaos for the tent map (Figs. 10,11), irrespective of criterions. Since $a = 1$ is the only edge, this result is expected.

For the logistic map, we have given a possible explanation to the priority of windows in Section

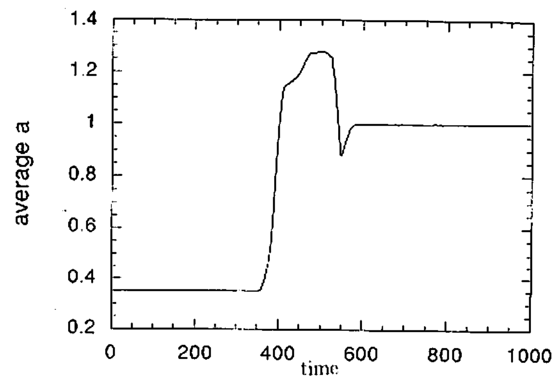

Fig. 10. Temporal evolution of the average tent parameter. The rule is given by [30 × 30 lattice, tent, Euclidean, replacement, 6.1×10^{-5}]. Initial values for the tent parameter a are chosen randomly from [0.3,0.4]. The deviation from 1 is very small after the equilibrium is established.

[2] Another way of weakening the difference (but keeping an analogue criterion) is the use of a modified definition for the distance measure; $D(1,2) = \Sigma_{m=1}^{T} |x_m(1) - x_m(2)|^{y}$, with $y < 2$. We have studied the case $y = 1$, which also enhances the fluctuations and the instability of the dominance of a species, similarly to the LR criterion. Such a modification may be rather superficial, however, as is immediately seen by considering the extreme limit $y \sim 0$.

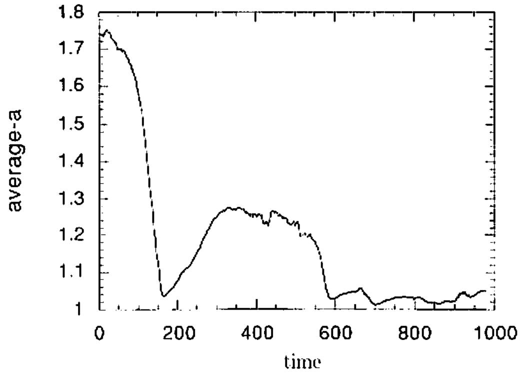

Fig. 11. Temporal evolution of the average tent parameter with the LR criterion. The rule is given by [30 × 30 lattice, tent, LR, replacement, 8×10^{-5}]. We have also simulated smaller μ cases, but the fluctuations remain still larger than those from the Euclidean criterion (given in Fig. 10).

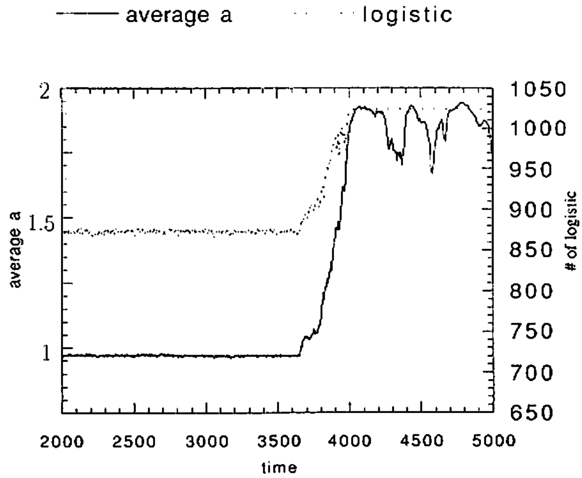

Fig. 12. Temporal evolution of the average value of the logistic parameters a (thin line) and the population of adopting a logistic map song. Here the lattice size is 32 × 32 (in total 1024 players) and $\mu = 7.5 \times 10^{-5}$. Around time step 4000, birds with logistic maps cover the total lattice and those with the tent maps are exterminated.

3. Is this edge of window stronger than the above onset of chaos in the tent map? Since no window exists in the tent map, it is interesting to study a system where birds with the logistic map and with the tent map can coexist and compete. Here we slightly modify the logistic map to

$$x_{n+1} = 2ax_n(1 - x_n) \qquad (5)$$

so that the variables (parameters a) in both maps take values in a same range [0,1] ([0,2]). We put players on a 2D square lattice, and adopt the criterion by a discrete set of symbols (L for $x < 0.5$ and R for $x > 0.5$). Initially, a bird's song takes either the logistic or the tent map randomly with equal weights. By starting from the population with small a values, the mean value of a for logistic birds evolves to a punctuated equilibrium value $a \sim 1$ after a few steps (see Fig. 12). At this equilibrium state, the ratio of the birds with the logistic map to those with the tent maps remains almost constant ~8:1. Then the logistic parameter shows an abrupt change to the value corresponding to the period-3 window (or the period-4 window, depending on system size). During the change, the birds with tent maps are completely exterminated, which means that the onset of chaos in the tent map is defeated by the windows in the logistic map. This result may serve as support for our hypothesis on the priority of the windows' edge.

5.2. Circle map

We have also studied the case with the use of the circle map

$$x_{n+1} = x_n + a\sin(2\pi x_n) + d\,, \quad \mathrm{mod}\ 1 \qquad (6)$$

as the song generator [13]. Now each bird possesses three parameters $a(i)$, $d(i)$, $\epsilon(i)$. We determine the rule for the 2-persons' game by measuring the cordial distance between two trajectories of maps as noted previously. Numerical results again show abrupt changes of parameters with successive punctuated equilibria. Synchronized temporal motions between average values of a and d are clearly seen in Fig. 13. The average of d seems to take an "equilibrium" value of either ~0 or ~0.5, resulting in a shift of the origin of the variables $\{x_n(i)\}$. On the other hand, the average a lies again at window

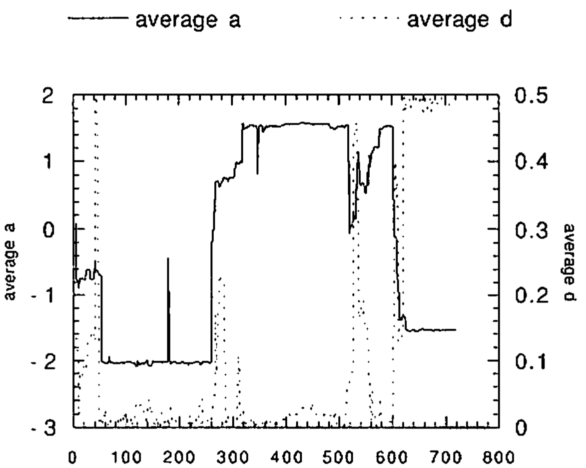

Fig. 13. Temporal evolution of the average value of a (thin line) and d (broken line) values for [30 × 30 lattice, circle, Euclidean, replacement, 5×10^{-5}].

values (to be precise, those with $d \sim 0$) $a \sim -2$, -1.5, 1.5, etc. [3].

Summarizing this section, the significant of the window structures in the imitation game, discussed in section 3, is further confirmed by changing the generator for birds' songs.

6. Dual dynamics

Before closing this report, let us present an example stressing clearly the aspect of dynamical complexity in the imitation game.

Let the logistic map be again the song generator. This time, however, we assign two parameter values for the logistic map to each player so that it can use a different value in imitating and in singing. This is not an irrelevant complication of the model. Rather, there is no a priori reason to believe that a bird should use an identical parameter for the two processes. For imitation games with one parameter, the restriction of using an identical parameter in singing and mimicking may be a cause to achieve an optimal value of either singing or mimicking, and lead to an edge. In the two-parameter game it may be possible that each process evolves to its own optimal value. Then one might expect that the results of the two-parameter games would be either of the following: (I) the parameters in singing and imitating evolve to the same "edge of chaos" value; or (II) the parameter in singing reaches a value for a fully chaotic state (~2.0) while the imitating value also evolves to a highly nonlinear region.

Surprisingly, the observed result supports neither of these. For the Euclidean criterion, the singing parameter again stays around $a \approx 1.94$, the edge of the period-4 window, while the imitating parameter stays around some other windows' edge (e.g., $a \approx 1.75$) or other bifurcation points (e.g., $a \approx 0.75$, bifurcation point from period-1 to -2), depending on the mutation rate and the topology of the game.

The results with the symbolic criterion are much more complex. Let us take an example with [2D lattice, logistic, LR, replacement, 5×10^{-5}]. The singing parameter switches among $a \sim 1.77$, 1.94, 2, in synchronization with the changes of the imitating parameter $a \sim 1.1$, 1.3, 1.77, (Fig. 14). This switching continues forever as far as we have checked.

The above result may be interpreted as follows: Optimal values for singing and imitating parameters are not unique, but there may be several local minima. Thus optimal values for the singing parameter depend strongly on the population distribution of the mimicking parameter at that time, and vice versa. A small disturbance is enough to change both the optimal values and the population distribution completely. Synchronized changes between the two parameters reflect such a subtle balance.

As stressed in Section 3, the fitness function is not given a priori but emerges through the

[3] To see the role of d, we also have carried out some simulations fixing $d = 0$ with same choices for other parameters. In this case, the magnitude of average a seems to be increasing monotonously with the punctuated equilibria at windows $\sim -5, -7, \ldots$. Thus d is not irrelevant, but plays a role in suppressing the magnitude of the temporal variation of a.

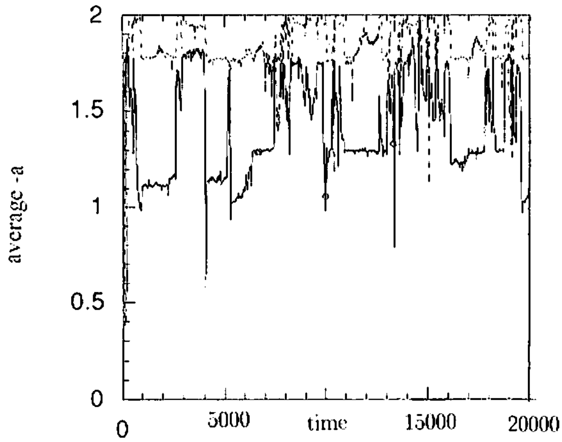

Fig. 14. Temporal evolution of the averages of the logistic parameters for singing (broken) and imitating (thin) in dual dynamics, with LR criterion. The lattice size is chosen to be 30×30, and $\mu = 7.5 \times 10^{-5}$.

evolution. For one-parameter games, however, the landscape is rather stable once the system evolves to the "edge of chaos". In the dual dynamics game, the landscape, besides its ruggedness, varies in quite a complex manner. Thus the model provides an illustration of a novel problem: evolution of a system with a dynamically changing rugged landscape.

Let us sum up the present section. The evolution to the edge between chaos and windows is still valid in dual dynamics, but the population dynamics is more complicated with successive alternations of dominant species and synchronized changes between the two parameters. This example illustrates that static (or "equilibrium") characterizations are not enough for imitation games, and that the understanding of the dynamics is essential.

7. Summary and discussions

In this report, we have examined the evolution of imitation games. Beside the confirmation of the evolution to the "edge of chaos" we have given a support to the priority of the edge of windows. The evolution to the edge of windows is commonly observed irrespective of topology of players, criterions for winners in a fight. It is also universally observed, independent of the choice of the map as a song generator (as is confirmed in the simulations with logistic and circle maps). We believe that our conclusion is universal for all dynamical systems (maps or differential equations) as a song, as long as they show chaos and windows with their parameter change.

The priority of the edge of windows is clearly demonstrated by the dominance of the window's edge of the logistic map over the onset of chaos in logistic and tent maps. Though birds with both maps can coexist with parameter values around the onset of chaos, the players with the tent map are completely exterminated with the abrupt change of the logistic parameter to the period-3 window's value.

Thus we propose an additional scenario here; *the evolution towards the edge between chaos and window*. This edge corresponds to the border between observable and invisible chaos. In a window, there is topological chaos in the dynamics. Although the final attractor is a periodic cycle for almost all initial conditions with a probability measure 1, there are chaotic orbits from nonmeasurable initial conditions (on a Cantor set). In connection with this topological chaos, there are chaotic transients before an orbit is attracted to the periodic cycle.

The existence of topological chaos assures a variety of unstable periodic orbits. Thus a player with a window parameter can imitate a large variety of periodic orbits, showing its dominance over players with periodic dynamics without topological chaos ($a < 1.4011\ldots$ in our logistic map). By the transient chaos, the dynamics has an ability to imitate chaotic time series roughly up to the length of transients, which diverges at the edge.

As a song generator, dynamics at a window parameter can provide a large variety of orbits as transients. Since the transient length diverges at the edge, a high variety of songs is maintained.

Thus generated songs are not easily imitated by periodic or chaotic dynamics.

Summing up, our *window's edge* scenario is based on the ability of creating complexity through topological and transient chaos. Transient chaos has a potentiality to adapt a wide range of external dynamics, while the orbit of the attractor is not complicated. Transient chaos may be important in a wide area of biological information processing, and our *window's edge* scenario may be applied universally in evolution and adaption.

Besides the characterization of the evolutionarily advantageous states, we have discussed the origin and maintenance of diversity of species, with representation of songs by a discrete set of symbols, and in dual dynamics for songs and mimicry.

If there is a given fixed fitness landscape, it would be reasonable to expect that the fittest species would dominate the world. Indeed, without any mutations or changes of the environment the diversity of species may not be sustained. There can be several possibilities for the origin of diversity. A simple answer may lie in the spatial differentiation mechanism. Indeed our simulation shows the coexistence of some species in a 2-dimensional topology. The diversity increases in the spatial game.

However, a more interesting discovery in the present paper is *symbolization induced diversity*. By adopting a criterion with a discrete set of symbols, the diversity is enhanced drastically. At some stage of information processing in the brain, representation by discrete symbols is often adopted. Thus we may expect that symbolization induced complexity can be one origin of the diversity in signals and languages [4].

For the criterion with discrete symbols, the population dynamics is also complicated. It shows successive punctuated equilibria forever. The complexity is further enhanced with the choice of dual dynamics for songs and imitations. A variety of window's edge states appear successively, providing temporal complexity. In a large system size, this complexity is easily expected to lead to spatial diversity, since the successive changes of dominant species cannot be synchronized over all lattice regions. Diversity induced by temporal chaos is recently discussed as the homeochaos scenario [14], where the population dynamics of species shows weak and high-dimensional chaos. Our window's edge scenario shares the notion of "weak chaos" with the homeochaos scenario. In homeochaos, we have also seen successive switches among ordered states, noted as chaotic itinerancy [15,16]. Detailed study of dynamical mechanisms of the successive switching in our case will be necessary in the future.

Our imitation game provides a universal route to the evolution to complexity. The pressure to escape from being imitated gives a trigger to the evolution to complexity. Conceptually, it can also be seen in many examples in biological or social evolution, besides our original motivation of a bird song. Such examples may include the evolution of a communication code ("secret code"), Batesian mimicry [17] and social structure [5].

Acknowledgements

The authors would like to thank Y. Iba, T. Ikegami, K. Tokita and S. Adachi for useful discussions, F. Willeboordse for critical reading of the manuscript. J.S. thanks Professor S. Asano for sharing CPU time on his work stations. The work is partially supported by Grant-in-Aids for Scientific Research from the Ministry of Education, Science and Culture of Japan.

---

[4] Did God destroy the Tower of Babel by giving us the ability of symbolization?

[5] For a related problem, with the use of control of chaotic dynamics instead of our imitation, see [18].

References

[1] N.H. Packard, in: Dynamic Patterns in Complex Systems, J. Kelso, A.J. Mandell, M.F. Shlesinger, eds. (World Scientific, Singapore, 1988) 293–301;
See, however, also the paper by M. Mitchell et al., Physica D 75 (1994) 361, these proceedings.

[2] C. Langton, Physica D 42 (1990) 12.

[3] S.A. Kaufmann and S. Johnson, J. Theor. Biol. 149 (1991) 467.

[4] K. Kaneko, Physica D 34 (1989) 1.

[5] K. Kaneko and J. Suzuki, Evolution to the edge of chaos in imitation game, in: Artificial Life III, C. Langton, ed. (Addison-Wesley, Reading, MA, 1993).

[6] C.K. Catchpole, Vocal Communication in Birds (Edward Arnold Press, Paris, 1979).

[7] J.R. Krebs, R. Ashcroft and M. Weber, Nature 271 (1978) 539–542.

[8] See e.g., S.A. Kaufmann and S. Levin, J. Theor. Biol. 128 (1987) 11.

[9] M. Mezard, G. Parisi and M.A. Virasoro, eds., Spin Glass Theory and Beyond (World Scientific, Singapore, 1988).

[10] R. May, Nature 26 (1976) 459;
P. Collet and J.P. Eckmann, Iterated Maps on the Intervals as Dynamical Systems (Birkhauser, Boston, Basel, Stuttgart, 1980).

[11] N. Eldredge and S.J. Gould, in: Models in Paleobiology, T.J.M. Schopf, ed. (Freeman, New York, 1972).

[12] K. Ikeda, K. Matsumoto and K. Ohtsuka, Prog. Theor. Phys. Suppl. 99 (1989) 295;
I. Tsuda, Chaotic neural networks and thesaurus, in: Neurocomputers and Attention, A.V. Holden and V.I. Kryukov, eds. (Manchester Univ. Press, Manchester, 1990);
K. Kaneko, Phys. Rev. Lett. 63 (1989) 219; Physica D 41 (1990) 137; 54 (1991) 5.

[13] K. Kaneko, Collapse of Tori and Genesis of Chaos in Disspative Systems (World, Scientific, Singapore, 1986).

[14] K. Kaneko and T. Ikegami, Physica D 56 (1992) 406–429.

[15] T. Ikegami and K. Kaneko, Chaos 2 (1992) 397–408.

[16] K. Kaneko, Physica D 75 (1994) 55, in these proceedings.

[17] J. Maynard Smith, Evolutionary Genetics (Oxford Univ. Press, Oxford, 1989).

[18] A. Pierre, A. Hübler and D. Pines, Physica D 75 (1994) 343, in these proceedings.