PATTERN COMPETITION INTERMITTENCY AND SELECTIVE FLICKER NOISE IN SPATIOTEMPORAL CHAOS

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For a coupled map lattice with a medium strength of nonlinearity, pattern selection and competition occur. Intermittent collapse of the selected pattern is found as the increase of nonlinearity. The intermittency is studied through the spatial and spatiotemporal power spectra. The latter show a flicker-like noise at low frequency only for modes of wavenumbers corresponding to the selected patterns. The scaling analysis with the change of the wavenumber and the window size is performed.

1. Introduction and model

Turbulent phenomena can be seen in a variety of systems, such as fluids, optics, solid-state physics, chemical reactions, liquid crystals, plasmas, and also in biology. One promising approach to such systems is to regard the turbulence as spatiotemporal chaos. For this approach the construction of a field theory of chaos based on a simple model is essential.

One of the most remarkable successes in recent field theory is lattice gauge theory [1]. Following the spirit of the theory, we have proposed a lattice chaos model using coupled maps [2–5].

A coupled map lattice is a dynamical system with a discrete time, discrete space, and continuous state [2–13]. As a simple and standard model for the spatiotemporal chaos, the following diffusive coupling model is used [2,4,6,10–15]:

$$x_{n+1}(i)=(1-\epsilon)f(x_n(i)) + \tfrac{1}{2}\epsilon[f(x_n(i+1))+f(x_n(i-1))] , \qquad (1)$$

where n is a discrete time step and i is a lattice point ($i=1, 2, ..., N$) with a periodic boundary condition. The function $f(x)$ is chosen to be the logistic map

$$f(x)=1-ax^2 ,$$

but the phenomena to be shown later can be seen in a wide class of mappings such as the circle map $x+A\sin(2\pi x)+D$.

As the nonlinearity a is increased, we have encountered with the pattern competition phenomena, i.e., patterns with certain wave numbers compete and a complex spatiotemporal structure is formed [14].

In the present letter, intermittency associated with the pattern competition is investigated. First we take the spatial power spectra $S(k)$ for $x_n(i)$. They show the coexistence of a peak at k_p and a broad-band noise at $k\approx 0$. Second, we take the space–time Fourier transformation of $x_n(i)$, the power spectra $P(k, \omega)$. It is found that they show an $\omega^{-\alpha}$ behavior at low frequency only for the modes with $k\approx k_p$. The dependence of the exponent α on the wavenumber k_p-k and on the nonlinearity parameter a is investigated. Lastly, a window is introduced to study the coherence in space and scaling behavior.

2. Intermittency by pattern competition (phenomenology)

As has already been reported [2–4,7], the model (1) shows period-doubling bifurcations of kink–antikinks to chaos as the nonlinearity parameter a is increased. If the nonlinearity is small, the kink structure itself does not move and the chaotic motion is

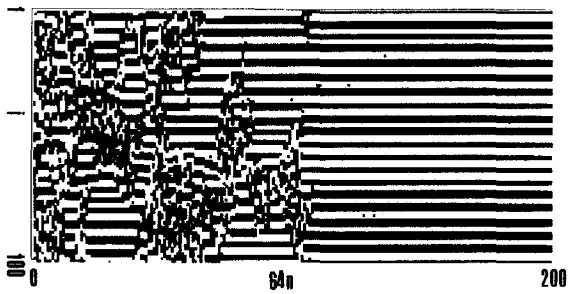
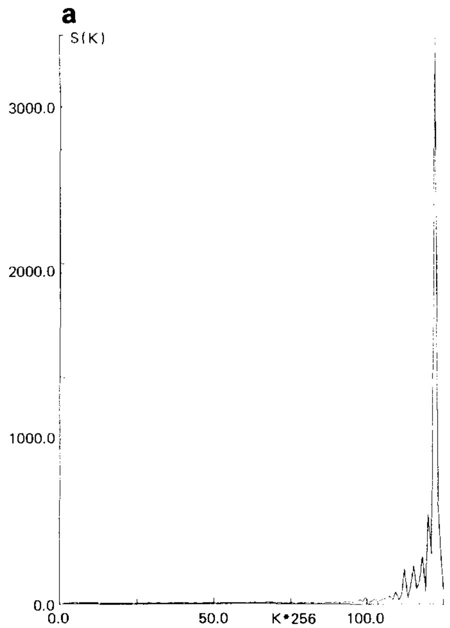


Fig. 1. Space–time diagram for the model (1), with $a=1.75$, $\epsilon=0.3$, $N=100$ and starting with a random initial condition. Every 64th time step is plotted from 0 to 200×64. If $x_{64n}(i)$ is larger than $x^*=(-1+\sqrt{1+4a})/2a$, the corresponding space–time pixel is painted as black, while it is left blank otherwise.

confined within each domain. As the nonlinearity is increased further, such domain boundaries start to move. Through the boundary motion, some spatial structures are selected, as is seen, e.g., in fig. 1. The statistics of the spatial pattern can be represented by the spatial power spectra defined by

$$S(k)=\langle\langle s(k,n)\rangle\rangle$$
$$=\left\langle\left\langle\left|\sum_{j=1}^{N} x_n(j)\,\mathrm{e}^{2\pi \mathrm{i}kj}\right|^2\right\rangle\right\rangle ,$$



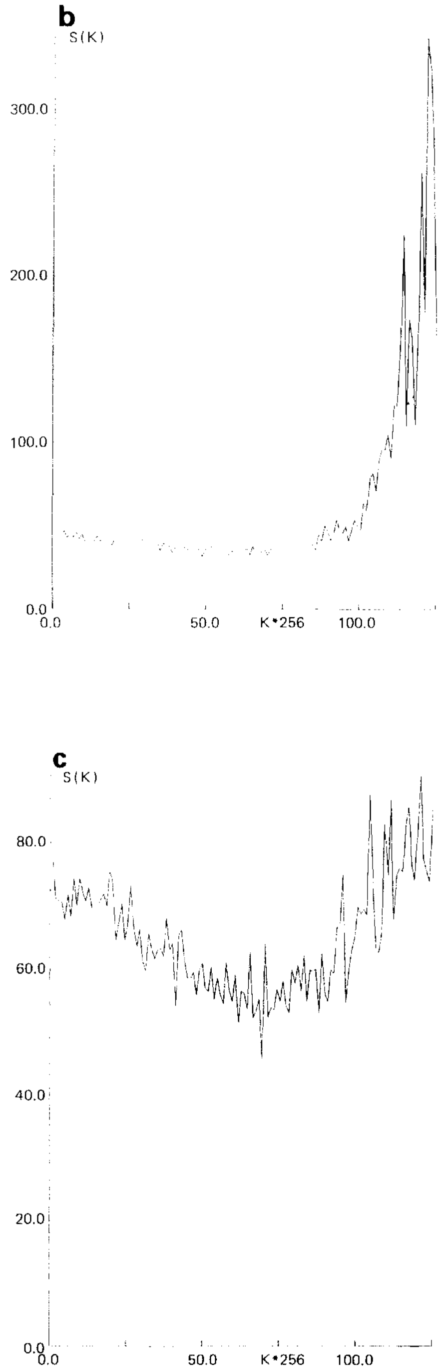


Fig. 2. Spatial power spectra $S(k)$ for the model (1). Random initial condition. Calculated from 1000 time step averages after discarding 10000 transients. (a) $a=1.85$. (b) $a=1.89$. (c) $a=1.98$ ($\epsilon=0.1$ and $N=256$)).

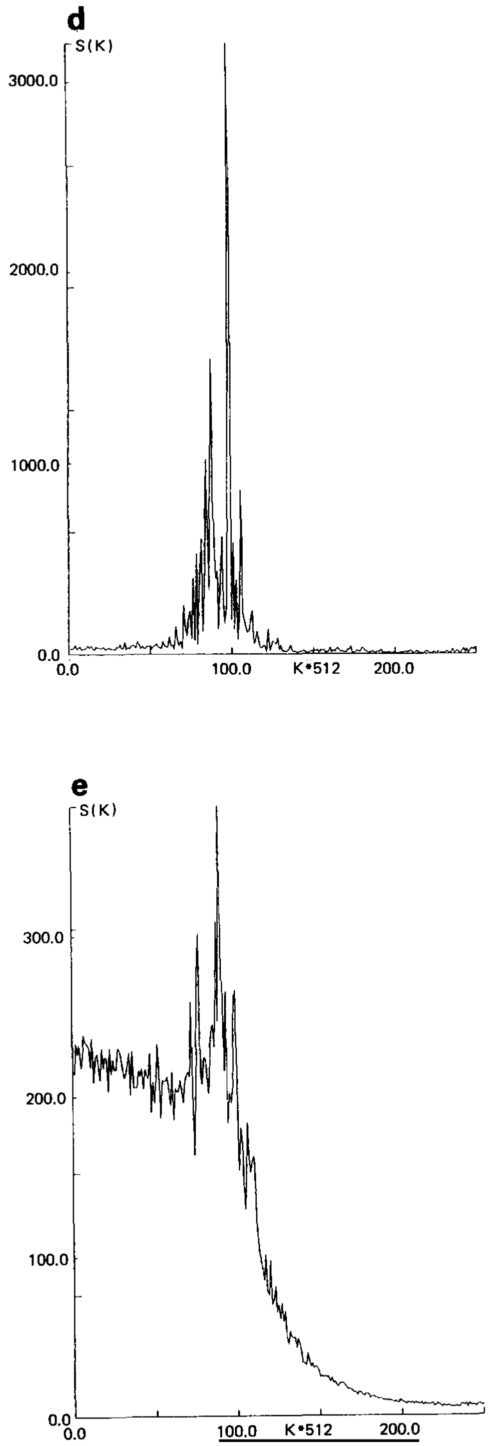


Fig. 2. Continued. (d) $a=1.73$. (e) $a=1.76$ ($\epsilon=0.3$ and $N=512$).

where $\langle\langle\,\rangle\rangle$ is the long time average after transients have decayed out. The spatial power spectra for $\epsilon=0.1$ are shown in fig. 2. The peak at $k_p=\frac{1}{2}$ is remarkable at $1.75<a<1.88$. In the real space, spatial zigzag patterns are self-organized. For larger nonlinearity ($1.88<a$) competition of the broad band at $k\approx 0$ (turbulent bursts) and $k\approx k_p=1/2$ (zigzag structure) can be seen. In these parameter regions, the zigzag pattern collapses in an intermittent way.

A similar kind of intermittency by pattern competition can be seen in a wide range of parameters for our model. A competition between the patterns with $k=k_{p1}=2/11$ and $k_{p2}=1/6$ are seen at $\epsilon=0.3$ for example, as can be seen in figs. 2d and 2e. In this case, the peaks in $S(k)$ at $k=k_{p1}$ and $k_{p2}\neq 0$ appear at some nonlinearity. As a is increased, broad band noise at $k\approx 0$ grows till the peaks at $k=k_p$ disappear. "Fully developed" turbulence is observed at larger a, where the power spectra are roughly approximated by $S(k)\propto\exp(-\text{const}\times Ak^2)$ [9] and have no prominent peaks at $k\neq 0$.

Here, the case with $\epsilon=0.1$ ($k_p=1/2$) is studied in detail, but similar phenomenology and arguments on the power spectra hold for other couplings ϵ with corresponding k_p's.

3. Power spectra for time and space

It is found numerically that the time series of the snapshot spatial power spectra $s(k, n)$ show a temporally intermittent behavior only for $k\approx k_p$ at the parameters for the pattern competition intermittency. This selective intermittent behavior is quantitatively studied by the spatiotemporal power spectra, i.e., the power of space–time Fourier transformation of $x_{2n}(i)$:

$$P(k,\omega)$$
$$=\left\langle\left\langle\left|\frac{1}{M}\sum_{j=1}^{M} x_{2n}(j)\exp(2\pi \mathrm{i}kj+2\pi\mathrm{i}\omega n)\right|^2\right\rangle\right\rangle .$$

Instead of taking the summation over all lattice points $j=1, ..., N$, we sometimes use a window, that is, we take a summation only over the restricted region, $j=1, 2, ..., M$ ($M\leqslant N$). Here, we show the case with $M=N$. Change with the window size is discussed in the next section. The results on the selective $\omega^{-\alpha}$

spectra are independent of the size of a window M. In order to remove the period-2 band like structure, the time step is taken alternately (x_{2n}).

Remark. Another way of calculation is a direct temporal power spectra $Q(k, \omega)$ for $s(k, 2n)$. Both of the results for $P(k, \omega)$ and $Q(k, \omega)$ give essentially the same results for the low-frequency phenomena.

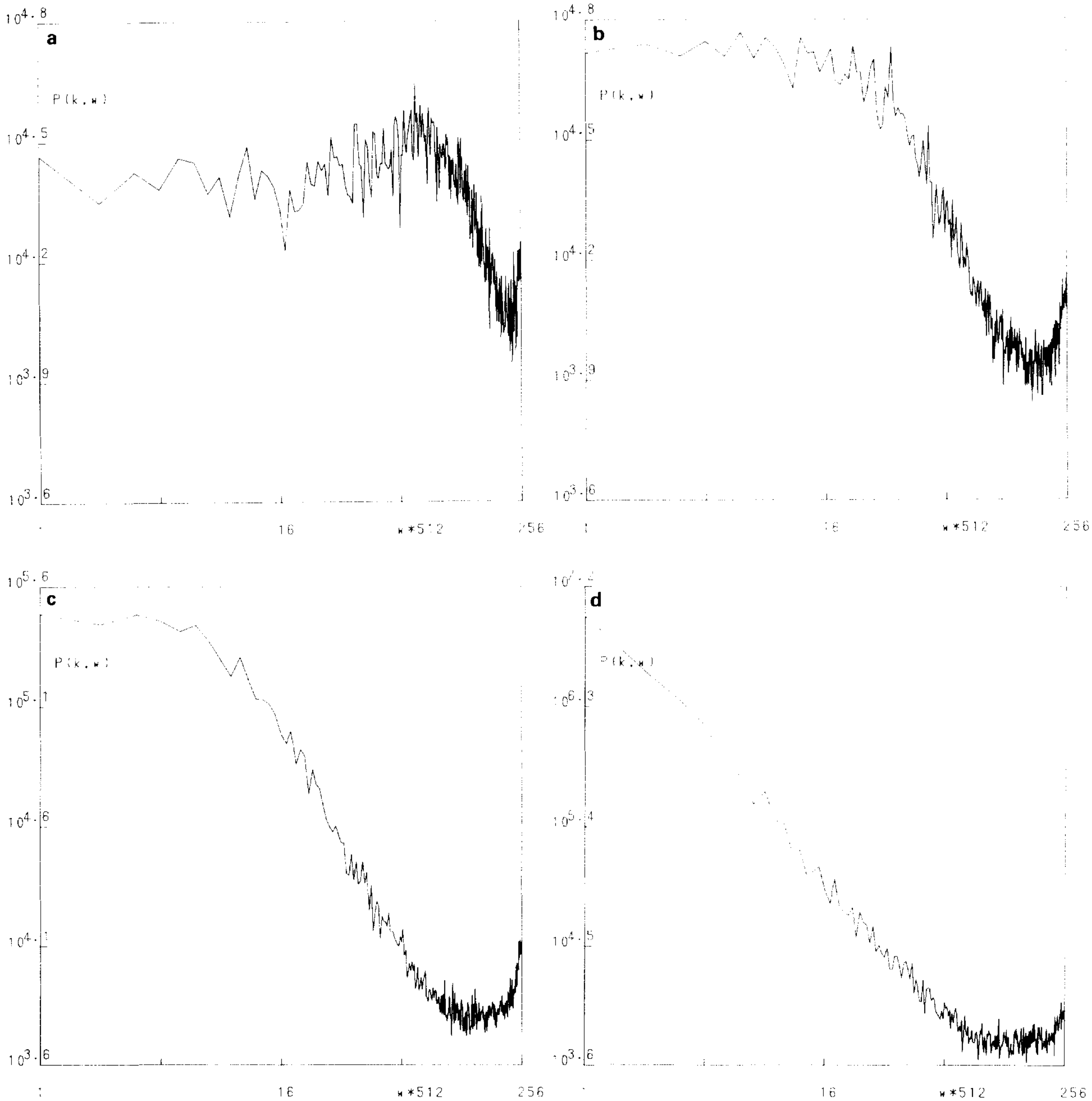


Fig. 3. Log–log plot of space–time power spectra $P(k, \omega)$ as a function of ω for $a=1.9$, $\epsilon=0.1$: (a) $k=0$, (b) $k=1/8$, (c) $k=3/8$. (d) $k=4/8$. The power spectra are calculated in the same way as in tables 1 and 2. The system size $N=256$ and the window size $M=256$.

Table 1
Low frequency exponents α as a function of bifurcation parameter a: The exponent α for $P(1/2, \omega) \sim \omega^{-\alpha}$ is estimated from the 512×2 time steps simulations of mode (1) with 50 ensembles, after 10000 steps of transients. Random initial condition. The system size $N=256$ and the window size $M=256$. For $a > 1.91$, the spectra have a plateau at very low frequency ($\omega < 1/32$) and the exponent α is estimated from the data at $1/32 < \omega < 1/4$.

a	1.88	1.885	1.89	1.895	1.90	1.905	1.91	1.915	1.92	1.93
α	1.99	1.89	1.86	1.76	1.72	1.64	1.64	1.64	1.64	1.65

At the parameter regions with pattern competition intermittency, our system shows the following flicker noise for the modes with $k \approx k_p$. In fig. 3, $P(k, \omega)$ is plotted for $k=0$, 2/8, 3/8, and 1/2. As k approaches 1/2, the low-frequency parts grow and $P(k_p, \omega) \approx \omega^{-\alpha}$ is clearly seen for $k=k_p$ ($=1/2$). Note that the flicker noise is selectively observed only for the modes $k \approx k_p$. For the pattern competition of two wavenumbers k_{p1} and k_{p2}, we have observed the flicker noise of $P(k, \omega)$ for $k \approx k_{p1}$ and $k \approx k_{p2}$.

At the onset of the collapse of the zigzag pattern ($a \approx 1.88$), α is close to 2, which means that the relaxation time diverges. As the nonlinearity parameter a is increased, the power α decreases from 2. The selective $\omega^{-\alpha}$ behavior holds for $k=1/2$ at $1.88 \lesssim a \lesssim 1.915$. Collapse of the zigzag pattern occurs more frequently for larger a, which leads to a faster decay of the correlation function and small α. The change of α with the parameter a is shown in table 1. For $a \gtrsim 1.915$, the power is roughly estimated from the data at $\omega \approx 1/10$, since they have plateaus at $\omega \approx 0$.

Next, we consider the change with wavenumber k. As k is decreased from $k=1/2$, the power decreases gradually., the exponents α are shown in table 1, where again, plateaus at $\omega \approx 0$ develop as k is decreased. For smaller k, the plateaus at $\omega \approx 0$ increases and the spectra approach the lorenzian form.

4. Window analysis

In the phase transition problem such as spin systems on a lattice, the notions of order parameter, scaling analysis, and renormalization group are powerful. To open the study of lattice theory of spatio-temporal chaos, it will be useful to develop these notions. The spatial power spectra $s(0, n)$ and $s(1/2,$

Table 2
Low frequency exponents α as a function of wavenumber k: The exponent α is estimated in the same way as in table 1. The system size $N=256$ and the window size $M=256$, $a=1.90$. For $k<15/32$, the spectra have plateaus at very low frequency ($\omega < 1/16$) and the exponents α are estimated from the data at $1/16 < \omega < 1/4$.

k	8/32	10/32	12/32	14/32	16/32
α	1.21	1.28	1.52	1.58	1.72

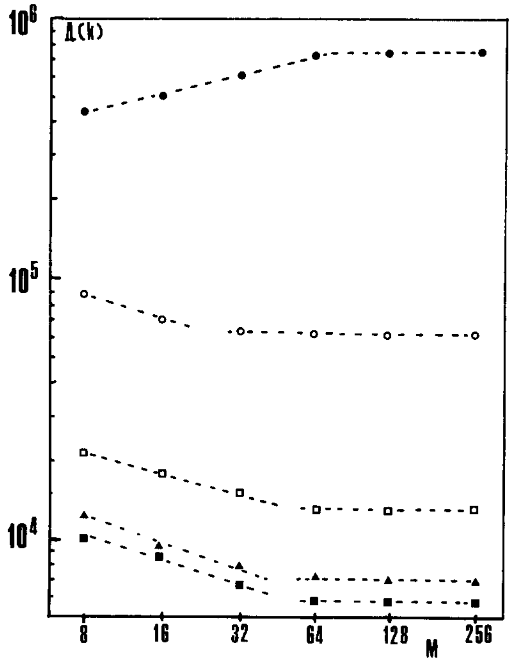


Fig. 4. The low frequency parts of $P(k, \omega)$ versus the window size M. $\Lambda(k) = P(k, 1/256) + P(k, 2/256) + \ldots + P(k, 8/256)$ are plotted versus the window size M. $a=1.90$ and $N=256$. ■ $k=0/8$, ▲ $k=1/8$, □ $k=2/8$, ○ $k=3/8$, and ● $k=4/8$. The power spectra are calculated in the same manner as in tables 1 and 2.

n) may be regarded as an extension of ferro and antiferro order parameters in spin systems. In this section we develop the scaling analysis for the dynamics of the order parameter with the use of a window [#1].

Let us focus on the power of some spectral mode $P(k, \omega)$. The spectral strength of such mode changes with the window size M. The strength approaches some constant with the increase of M, if the correlation length is finite and the system attains some local equilibrium. Near the critical point of the pattern competition intermittency, the spatial correlation length can be very large, and the spectral strength can change as $M^{\beta(k)}$ for $M<M_c$ and approaches a constant for $M>M_c$.

The numerical results for the low-frequency spectral strength Λ are shown in fig. 4. Here the strength Λ is estimated as

$$\sum_{i=1}^{8} P(k, \omega_i = i/1024).$$

We note that $\Lambda(k)$ decreases as $M^{\beta(k)}$ ($\beta(k)<0$) for $k=0$, 1/8, and 2/8 as M is increased. The decrease stops at $M\approx M_c$ (≈ 64) and stays constant for $M>M_c$. On the other hand, Λ is increasing as $M^{\beta(k)}$ ($\beta>0$) for $k=1/2$ up to $M\approx M_c$. We note that $\beta(k)$ is positive only for $k\approx k_c$, while it is negative for other modes.

The above scaling behavior is expected to hold up to $M\to\infty$ at the critical point, if we believe in the knowledge of phase transition studies in spin models. Our results near the transition show a crossover from $M^{\beta(k)}$ to a constant at $M\approx M_c$.

Taking the results of scaling into account, we may conclude that the mode with $k\approx k_p$ is relevant in the sense that the power increases with the size of the window, and other modes are irrelevant at the critical point.

At $a<1.88$, the chaotic burst corresponding to $k=0$ is transient and a single zigzag pattern covers the whole space as $n\to\infty$. If we perform the window analysis at the transient time regime where the chaotic bursts still exist, the M^{β}-behavior is observed without the crossover. For $k=1/2$, β is positive, while it is negative for other modes. This shows the nonstationary feature of the turbulence and (ir)relevancy of modes clearly.

[#1] For a similar approach to a discrete epidimeological model, see ref. [16].

5. Summary and discussions

We have reported a new type of robust intermittency. It arises from pattern competition. A selected pattern with $k=k_p$ does not remain stable and collapses intermittently through bursts. This type of behavior may be observed in other nonlinear problems, e.g., in fluid experiments, where the pattern competition is observed [17,18].

We have introduced a kind of filtered power spectra study. The essential point is that the flicker-noise appears only for modes $k\approx k_p$. The filtered power spectra study for the experiments in the pattern competition is recommended.

Filtered analysis on the power spectra has been developed in the fluid turbulence problems, which may bring about fruitful results in the nonlinear field theory problems [19]. The selectivity by the modes we observed may be also seen in other problems such as the brain wave.

The present intermittency seems to belong to a different class from the Pomeau–Manneville one [#2] [20,21] and to be closely related with another mechanism suggested by Crutchfield [22,10] [#3]. The $\omega^{-\alpha}$ behavior is widely seen in intermittency problems [23]. The important difference here is the robustness and selectivity to wavenumbers in our case.

The change of relevancy with the wavenumber strongly reminds us of the renormalization group analysis in the phase transition problems [24]. The scaling property with filtered power spectra and window analysis may be theoretically formulated through some kind of renormalization group analysis.

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[#2] For spatial intermittency of Pomeau–Manneville type see ref. [5].

[#3] In ref. [10] a new type of intermittency is reported, which is closely related to ours.

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