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Dynamical systems game theory II
A new approach to the problem of the social dilemma

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Abstract

The "social dilemma" is a problem inherent in forming and maintaining cooperation among selfish individuals, and is of fundamental importance in the biological and social sciences. From the viewpoint of traditional game theory, the existence of the social dilemma necessarily implies degeneration into selfish behavior as the numbers of members in a community increases, unless there exists some external power. In the real world, however, cooperation is often formed and maintained merely through mutual interactions, without the influence of an external power. To answer questions concerning appearance and maintenance of cooperative behavior in societies, we study what we call the "Lumberjacks' Dilemma (LD) game", as an application of the dynamical systems (DS) game theory presented in [Physica D 147 (2000) 221], which can naturally deal with the dynamic aspects of games. Dynamical processes that lead to the formation and maintenance of cooperation, which is often observed in the real communities, are realized in our model. The mechanism underlying this formation and maintenance is explained from the DS game point of view, by analyzing the functional dependence of the attractor of the game dynamics on a parameter characterizing the strategy. It is demonstrated that norms for cooperation are formed as strategies that are manifested as specific attractors of game dynamics. The change in the stability of this cooperative behavior as the number of members increases is also discussed. Finally, the relevance of our study to cooperation seen in the real world is discussed.

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1. Social dilemma

Constructing and maintaining cooperation within a relatively large group of individuals often entails a dilemma, as is seen, for example, in the problem of garbage disposal, where those who do not care about the public good can attain a relatively high utility. More generally, this type of dilemma exists in the situation that the collective profit of the entire community is maximized by cooperation, but each member individually can obtain a larger personal profit by behaving selfishly. In this case, theoretically, rational players should behave selfishly. However, if they do so, the society will be damaged and will eventually fall apart. Here, rational behaviors of the community members

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paradoxically bring about a bad result. The problem involved in the maintenance of cooperation in a social group is generally called the "social dilemma", for which the *n*-person Prisoners' Dilemma game is often used as a typical model. The social dilemma is important in sociology, socio-biology and other fields.

1.1. The Tragedy of the Commons

Let us briefly review some studies of the social dilemma [18].

The classical story expressing the social dilemma is the so-called "Tragedy of the Commons", presented by Hardin in 1968 [12]. Because of its applicability to a variety of environmental issues, this story has been often referred in the fields of sociology and political science. The story is the following:

> "There is a pastureland open to any villager. If the villagers are completely free to put their cattle out to graze, and if there is no restriction placed on each person's use of the land, they always increase the number of cattle they send to pasture to increase their personal profits. As a result, the common pasture is eventually exhausted, and pasturage becomes impossible. The loss by increasing one cattle to the pasture is not large, but if this increase continues, the capacity for breeding cattle will eventually collapse."

An example of the Tragedy of the Commons is provided by the collapse of grazing land in North America [19]. The total area of pastureland in North America is 312 million ha, which occupies one-third of the entire area of the US. On most of this area, domestic animals are put out to pasture, and the cattle consume about 95% of the grass. In the late 19th century, ranchers in the west continually increased the number of cattle, and left them without control. As a consequence, most of the highlands became stripped, the soil became poor, and eventually pasturage for cattle became almost impossible. Since this tragedy, grazing land has gradually been recovered by mutual cooperation among community members, by dividing the land into parts and using them in turn.

Table 1
The payoff matrix for the Prisoner's Dilemma[a]

Player 1	Player 2	
	C	D
Cooperate	(3, 3)	(0, 5)
Defect	(5, 0)	(1, 1)

[a] In the Prisoner's Dilemma game, the two players either cooperate or defect (act selfishly). For each element of this table (S_1, S_2), S_1 is the score of player 1 and S_2 that of player 2.

1.2. Theoretical works on the social dilemma

In socio-biology, the emergence and maintenance of cooperation in society is thought to be a result of kin selection [11] or altruistic reciprocity. The first important study of cooperation based on "reciprocity" was the experimental and theoretical study of the iterated Prisoners' Dilemma game carried out by Axelrod [2]. (The payoff matrix of the Prisoners' Dilemma is shown in Table 1.) He showed that the simple strategy TIT-FOR-TAT is evolutionarily stable against other strategies, such as ALL DEFECT, and as a result, a cooperative society can be maintained.

This explanation of the appearance and maintenance of cooperation based on the iterated Prisoners' Dilemma has been applied to a variety of social phenomena. However, many researchers have come to believe that a direct application of the result of the iterated Prisoners' Dilemma to the problem of cooperation in a group is difficult, because interactions in a society usually involve more than two individuals. Therefore, the necessity of game models with more than two players has been recognized. For example, Axelrod and Dion argued that the social dilemma should be formulated as an *n*-person Prisoners' Dilemma with $n > 2$ [4].

Boyd and Richerson [5] and Joshi [14] have analyzed the *n*-person Prisoners' Dilemma using evolutionary games, and proved that the condition for (the *n*-person version) TIT-FOR-TAT to be evolutionary stable is harder to be satisfied as n increases. Let us call this *the effect of the number of players*. In this case, 'reciprocity' is not sufficient to explain the maintenance of cooperation. Hence, the following two questions are raised:

(1) If reciprocity is not sufficient to explain the maintenance of cooperation in a social group, how is cooperation maintained?
(2) Is the n-person Prisoners' Dilemma really appropriate to study the social dilemma?

With respect to the first question, Boyd and Richerson have found an additional strategy, "sanction" against non-cooperators in the n-person Prisoners' Dilemma. They have also investigated the evolution of sanction (see [7], for example). Axelrod has also introduced a model that involves a 'metanorm', which involves an incentive to punish not only betrayers but also those who have not punished betrayers [3]. Following another line, Boyd and Richerson considered a cultural effect to maintain cooperation [6]. These works are concerned with the way that cooperation in a society can be maintained by external factors other than mere reciprocity between agents. Actually, as discussed in the field of human ecology, "tragedy" in real society is often avoided through the action of the institutions outside the interacting agents.

Regarding the second question posed above, a problem involved in using the n-person Prisoner's Dilemma to study the Tragedy of the Commons has recently been pointed out [9]. Indeed, an appropriate description for the 'tragedy' may not be the Prisoners' Dilemma, but the *Chicken game*. (The payoff matrix of the Chicken game is given in Table 2.) In the Chicken game, the simultaneous selfish behavior of both players results in the lowest overall value of the wellbeing, while in the Prisoners' Dilemma this results in the second lowest such value. In this regard, the Chicken game appears to be more realistic, because in the real world, with the continuous use of a common resource, the overall wellbeing is lowest when all members waste this resource and suffer bankruptcy.

Table 2
The payoff matrix for the Chicken game[a]

Player 1	Player 2	
	C	D
Cooperate	(3, 3)	(1, 4)
Defect	(4, 1)	(0, 0)

[a] In the Chicken game, two players either cooperate or act selfishly.

Summing up, there is a recent trend in thinking that "reciprocity" is not sufficient for the maintenance of cooperation in a society of interacting individuals, and that sanction or some other strategies, based on institutions or norms, are necessary.

1.3. Problems in modeling the social dilemma

Here we discuss two problems associated with the model study of the social dilemma. The first problem concerns the use of an external norm, and the second problem involves the disappearance of the dilemma itself.

(1) As discussed in the last subsection, the introduction of external social norms is often made in studies of the social dilemma. However, if these norms come from a source outside of the interacting agents (for example, from a government), we cannot study what type of norms will emerge spontaneously in a society. For example, after a cooperative social norm is set, the success in creating a cooperative society depends on how many individuals sacrifice themselves for the society. If such a norm completely ignores personal benefit, it is unlikely that people will obey it. People will only conform to norms that allow some personal benefit. As long as the norm is externally set in models, however, we cannot discuss what norm could be selected spontaneously by players.

Another question regards the origin of external norms. Do people always need some external institution, like a government, outside of their community? Of course, there is no doubt that people cannot avoid tragedy if they cannot communicate with each other, for example, when they live in a large village. However, this does not necessarily imply the need for a government, because a norm may be formed through the players' communications or interactions alone, without the presence of an external institution. For example, negotiations or struggles among nations may sometimes result in cooperation, even without an external "metapower". (Only an exception for this is the cooperation formed through the occasional intervention of the UN.) Furthermore, even if norms and sanctions applied from outside the system are necessary in some cases, it may be possible to create a social structure maintained only by the individuals within the system. If this is true, such social structure would certainly be more stable than structure that is simply imposed on society only by some external force. Then, it is important to ask if there is some characteristic feature of such a social structure.

(2) The inclusion of norms and sanction strategies changes the nature of game itself. It is not necessarily true that the original dilemma still exists in this modified game environment. To see this, consider the following. Assume that each player has a strategy to sanction other players who act selfishly. If it is easy to apply sanctions to players who do not conform with the norm, it is a matter of course that the game society becomes cooperative. That is, if the cost for punishing the betrayers is small, such sanctions will be advantageous in the long run, since they will eliminate selfish behavior, and a cooperative society will be realized. In this case, however, it is not correct to conclude that "a cooperative society has emerged in the presence of the social dilemma". Rather, one should better say that "cooperation resulted because the dilemma has been eliminated as a result of the change of the rules". In fact, if the rules of the game explicitly permit the punishment of betrayers, this game differs from those with a social dilemma at the level of a payoff matrix.

From the viewpoint of sociology, it is surely interesting to work out how to introduce additional rules, such as sanction and norms, in order to change the game and dispel its dilemma. However, the problem we would like to deal with here is that of the spontaneous formation and maintenance of cooperation *within interacting players* in the presence of this dilemma.

Without an external institution, it is true that the tragedy of betraying each other may sometimes arise. However, it is also true that we do not always fall in the tragedy and that we can sometimes cooperate in a community, even under the social dilemma, by creating tacit consensus, some kinds of ritualizations, mores, and so forth, which emerge through mutual communication and interaction. To understand this, it is necessary to consider how cooperation is organized in the presence of this dilemma. In this case, what we should study is not a system in which the changes of the rules of the game are made externally, but the system in which they are formed spontaneously through interaction of players.

1.4. Dynamical structure of games

Here we discuss some important characteristics of social dilemma in the real world that are not (or cannot be) treated by models in the traditional game framework.

Most of the real systems plagued by the social dilemma, whether they end in tragedy or not, can basically be understood in terms of some characteristic dynamics, such as decrease of petroleum resources, fluctuations of livestock resources, marine resources and a change of peoples' economic conditions. It is often the case with such systems that comprehension of the space–time structure of the systems plays an important role in the avoidance of tragedy [13]. For example, in the situation involving grazing land in North America discussed above, and in the open field of Medieval Europe, where the so-called three field system of crop rotation was widely used, tragedy was avoided through cooperation based on the temporal and spatial differentiation of their roles that take into account the geographical nature and the innate dynamics of the resources. The cooperative dynamics actually observed in these examples correspond to attractors that emerged according to some internal dynamical laws and the decisions made by people that were part of the system. Issues concerning, for example, the stability of such cooperative attractors, which are made by the strategies of the people, against the invasions of other strategies cannot be addressed without consideration of the dynamical structure of the systems in question. In these games, from the players' point of view, in order for players to cooperate, they sometimes have to decide themselves what the norm (the consensus about what level of behavior can be considered cooperative) is, if there is no outside agent who gives them the norm, and such self-determined norms can not be considered apart from the concept of ‘time’ or ‘dynamics’. For example, if the current prevalent behavioral pattern maintained by a cooperative player group makes the dynamics of the game environment in a way that is relatively productive for all players, then no player will violate the cooperative behavioral norm, because such violation might give rise to a ‘change’ of the game environment into a nonproductive state in the future, in which all players, including the violator, have a lower wellbeing. In such cases, *the robust norm* may be found only in the dynamical structure of games.

It is certainly productive and of course important to represent various situations involving social dilemma in a traditional (static) way, that is, as a payoff matrix of n-person Prisoners’ Dilemma. Thus, we can make clear an important feature of the games. However, representations games in a static form alone may entail the loss of features of specific dynamics in the games. In the present paper we study the social dilemma from a dynamical and novel point of view, by employing *dynamical systems (DS) games*, where the game is conducted repeatedly (like the iterated Prisoners’ Dilemma game), while the nature of the game can change with time through the effect of the players’ actions.

2. DS game

In the real world, when we decide to select an action and carry it out, our behavior sometimes changes our own game environment. Moreover, a change in the environment may also have an effect on a person’s decision-making process. Further, the utility of a given behavior sometimes varies according to that individual’s (or others’) current circumstances (for example, the pleasure we find in some action sometimes decreases as we get tired of it). However, the static description provided by traditional game theory, whether it is in the form of a payoff matrix or a game tree, is not suited to describe and treat this type of dynamical phenomena. In order to model such phenomena, we presented DS games [1]. In a DS game, the game itself can be affected and changed by the players’ behavior and states. In other words, the nature of the game itself is described as a ‘DS’. Here we review briefly the basics of the DS game framework. The details of this formulation and discussion about DS games are given in [1].

In a DS game, players live in a certain game environment and choose among several possible actions. The game dynamics, g, are composed of the following three component process:

(1) The states of the players’ surroundings (which we call the *game environment*), x, and those of all the players, y, change following a ‘natural law’ represented by some equation of motion.
(2) Players make decisions according to their own decision-making mechanisms, f, by referring to both the states of the game environment and of all the players (including themselves).
(3) A change in the game environment and the players’ actions affect the states of the players.

The game dynamics with the above components is continuously repeated, and the nature of the game itself can change through this repetition of g:

$$g : (x(t), y(t)) \mapsto (x(t+1), y(t+1)).$$

The entire game dynamics are described by this map g, and the players’ decision-making functions, f, are subsumed within g. In this way, a DS game explicitly describes the game-like interactions as well as the dynamics in the game.[1]

3. Lumberjacks’ Dilemma (LD) game

As an application of the DS game framework to the social dilemma, in this paper we use the LD game. Here we summarize the LD game, whose detailed formulation is given in [1].

[1] In fact, the game dynamics could be described either in a continuous-time style (by using differential equations) or a discrete-time style. We use the latter in this work, since it is convenient for comparison with the so-called “iterated game model” of traditional game theory, which consists of a mere repetition of a static payoff matrix.

3.1. Abstract description of the LD

Let us consider the following story that describes the situation of LD game:

> "There is a wooded hill and several lumberjacks on it. The lumberjacks fell trees for a living. They can maximize their collective profit if they cooperate in waiting until the trees have grown fully before felling them, and share the profits. However, any lumberjack who fells a tree earlier takes the entire profit on that tree. Thus, each lumberjack can maximize his personal profit by cutting trees earlier. If all the lumberjacks do this, however, the hill will become barren and there will eventually be no profit. This situation represents a dilemma."

The LD game provides an example of the social dilemma. In other words, it can be represented in the form of an (*n*-person) Prisoners' Dilemma if we project it onto the space of static games. However, there are several important differences between the LD game and the *n*-person Prisoners' Dilemma. First, dynamics of the size of the trees are expressed explicitly in this LD game. Also, the yield from one tree, and thus a lumberjacks' profit upon felling it, differs according to the time that it is felled. These profits have a continuous distribution, because the yield of a tree takes continuous values. Finally, a lumberjack's decision today can affect the future game environment due to the dynamic nature (i.e. the growth) of trees.

3.2. Modeling

3.2.1. The game-world of the LD game

We now discuss the concrete modeling of the LD game. In the game-world, the ecology of the LD game, there are h wooded hills in which the lumberjacks live. Suppose that the lumberjacks in the population can be classified into s species. We define the set of the 'lumberjack species' as $S = \{1, 2, \dots, s\}$ and the set of the hills as $H = \{1, 2, \dots, h\}$. The lumberjacks who belong to a species i ($\in S$) have the same decision-making function f^i, and therefore they all exhibit the same decision-making behavior. Each lumberjack belongs to one hill and competes with the other lumberjacks that belong to that hill to fell trees that grow in time, according to his strategy. The game played on each hill is played completely isolated from the games played on the other hills; that is, the lumberjacks on a given hill have no idea how the games are progressing on other hills. (In all the simulations, s is 10 and h is 60.)

Let us denote the number of trees on each hill by m, and that of lumberjacks by n. Now, on each hill, n lumberjacks (players) compete over m trees (the resources of the hill) to be hewed as lumber. In the computer experiments carried out in the present study, the n lumberjacks who live on a given hill were selected randomly from the s species, and multiple lumberjacks of the same species can live on the same hill. We define the set of players on a hill as $N = \{1, 2, \dots, n\}$ and the set of resources as $\Xi = \{1, 2, \dots, m\}$. These n players play a 400 round repeated game ("the LD game"). At each round, each player has a 'score' of his wellbeing according to the value of the lumber he cuts in that round. After $t = T$ rounds, each player's average score over these T rounds is calculated. (In the simulations in this study, T was set to 400 rounds.)

This LD game is played parallely over T rounds in each of the h hills, which gives *one generation* of the game. The *fitness* of a species is defined by *the average of the average scores* that all the players of that species have acquired on all the hills. Before the next generation of players enters the game, the k species with the lowest values of fitness are replaced by k new species. These new species are obtained as mutants of the k species randomly selected from among the other $(s-k)$ species. The other $(s-k)$ species survive for the next generation. The same procedure is repeated in the next generation, without the memory of the previous generation. (Throughout all experiments in this study, the parameters s and k were set to $s = 10$ and $k = 3$.)

3.2.2. LD game conducted on a hill

The LD game played by n lumberjacks (players) on h hills competing for m trees (resources) is described as follows:

Let us denote the 'state' of the resources on a given hill at time t by $x(t)$. This is a vector-valued function whose elements are the 'heights' of the m trees; $x(t) = (x_1(t), x_2(t), \dots, x_m(t))$. Each player possesses a one-dimensional variable that represents his 'state'. Here, a 'state' represents, for example, the monetary state, nutritional state, and so forth, of the player. Its value at any given time represents that player's score for that round. Each player also has his own decision-making function. The state of the player i is denoted by $y^i(t)$ and his decision-making function by $f^{S(i)}$ where $S(i)$ is his species in the game-world. We define $y(t)$ and f as $y(t) = (y^1(t), y^2(t), \dots, y^n(t))$ and $f = (f^{S(1)}, f^{S(2)}, \dots, f^{S(n)})$.

At any given time, each player decides his next action by considering the sizes of the trees, $x(t)$, and the states of players, $y(t)$. All of the players' actions are represented by the vector $a = (a^1(t), a^2(t), \dots, a^n(t))$. Each player's individual action at a given time can be one of $m + 1$ actions: "do nothing", "cut tree 1", "cut tree 2", ..., "cut tree m". These actions are represented by $0, 1, 2, \dots, m$, respectively, and the set of all these feasible actions, $\{0, 1, 2, \dots, m\}$, is denoted by A.

The T-times repetition of the map g generates the game dynamics $((x(t+1), y(t+1)) = g(x(t), y(t)))$. In the simulation of this study, for the first round of the game on each hill, the values x_i ($i \in \Xi$) were all set to 0.1 (although this choice itself is not so important since soon the resource dynamics falls on an attractor independent of the initial values). The players' initial states y_j ($j \in N$) were chosen as random numbers from the normal distribution with mean 0.1 and variance 0.1. The map g is composed of three components: (1) natural law, (2) decision making by players (f) and (3) effects of actions:

(1) The natural law is one of the components determining the overall dynamics of the game, but has nothing to do with the decision making of players. In the LD game, the natural law consists of (i) the growth of the trees, $x_k(t)' = u_\Xi(x_k(t))$ $(k \in \Xi)$, and (ii) the decrease of the values of players' states, $y^i(t)' = u_N(y^i(t))$ $(i \in N)$. In this study, we set $uN(y) = 0.8y$. (The formula for u_Ξ is given below.)
(2) Player i's decision-making function, $f^{S(i)}$, *determines his action*, $a^i(t)$, based on the states of the environment and the players on the same hill, denoted by $x(t)'$ and $y(t)'$. In other words, $a^i(t) = f^{S(i)}(x(t)', y(t)')$. The term $f^{S(i)}$, which varies throughout the evolution, represents 'internal structure' of the player i and is invisible to other players.
(3) Players' actions *affect the state of the resource* on the hill. The height of tree i when cut by the players decreases according to $x_i(t + 1) = (1/3)^{\nu_i} x_i(t)'$, where ν_i is the number of players who cut this tree. The lumber cut from any given tree is divided equally among all the players who cut that tree. Acquiring lumber in this way increases the value of the players' states. Thus, player i's state becomes $y^i(t+1) = y^i(t)' + \Delta$ in the subsequent round, where Δ is the amount of lumber this player received in the present round.

These three steps as a whole is called *one round of the game*. In each round, for any player to increase his score in that round, it is always better to cut a tree, because Δ is always positive if he cuts a tree, while it is zero if he chooses to 'wait'. However, in the long run, taking action of 'cutting' too frequently does not always result in the high *average* score.

3.2.3. Settings for LD games

3.2.3.1. Two natural laws of the tree growth.

In this paper, we consider two types of maps to describe the natural law of the tree growth, u_Ξ:

(1) $u_{\Xi_C}(x) = 0.7x^3 - 2.4x^2 + 2.7x$,
(2) $u_{\Xi_L}(x) = \min(1.5x, 1.0)$.

We call u_{Ξ_C} (Fig. 1(a)) the *convex map* and u_{Ξ_L} (Fig. 1(b)) the (*piecewise*) *linear map* because of their graph shapes. Here the detailed forms of these maps are not important. The point is that we have defined two *different* natural laws with *a common nature*. For example, in either case, a tree grows rapidly during the early rounds, but its growth gradually slows and its height eventually converges to 1 if it is never cut. Furthermore, both natural laws bring about a LD, because players can maximize their collective profits through cooperation by waiting for trees to have fully grown before felling them, while any individual player can

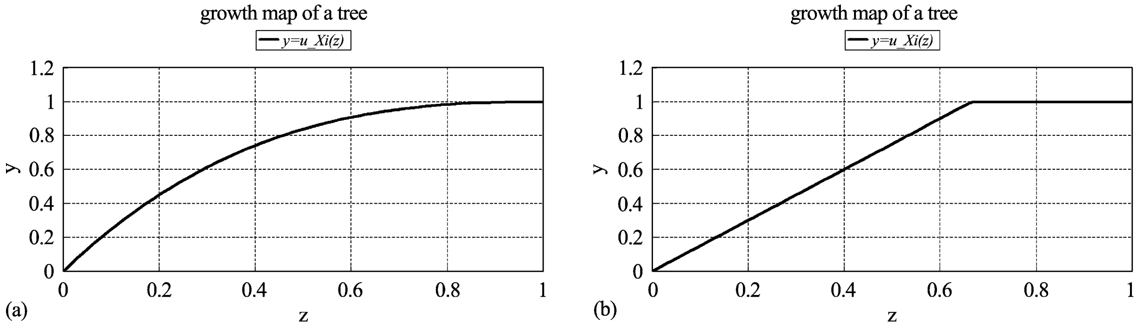

Fig. 1. Maps for the growth of trees, u_{Ξ}: (a) convex map; (b) linear map.

receive the entire profit from a tree if he alone does not cooperate and cuts the tree before maturity. The situations of the both the convex and linear LD games would be represented by the same payoff matrix of the *n*-person Prisoners' Dilemma in the traditional, static game framework.

3.2.3.2. Decision-making function. To implement concretely the decision-making function *f*, we introduce the "motivation map" mtv_r for each feasible action r ($r \in A$), which gives a player's incentive to take the action r. The values of the coefficients of mtv for a player change as this player evolves. The form of this map as follows:

$$\text{mtv}_r : (x, y) \mapsto \sum_{k \in M} \eta_{kr} x_k + \sum_{l \in N} \theta_{lr} y^l + \xi_r.$$

Here, the values η_{kr} and θ_{lr} form real number matrices and the values ξ_r form a real number vector. These coefficients, which change through mutation, determine the player's strategy, and a tiny difference in these values can lead to those different strategies for which 'fitness' clearly differs.

In each round, each player selects the action whose motivation has the largest value among the set $\{\text{mtv}_r\}$.[2]

The coefficient parameters η_{kr}, θ_{lr}, and ξ_r of the initial 10 lumberjack species in the game-world are generated as random numbers from the normal distribution with mean 0 and variance 0.1. The coefficients of offspring are chosen as random numbers from normal distributions with variance σ, and means equal to the corresponding parameters of the parents. The value σ, corresponding to the mutation rate, is set to 0.1 here.

[2] The function $f(x, y)$ is given by

$$f(x, y) = k \; (k \in A) \text{ if } \text{mtv}_k(x, y) \geq \max_{r \in A} \text{mtv}_r(x, y).$$

4. Evolutionary LD games—effect of the number of players and game dynamics

In this section, we briefly explain how the number of game participants and the game dynamics influence the evolutionary phenomena seen in the games. As stated in the previous sections, it is already known from the analytical findings of traditional, static games that under social dilemma, it is harder to achieve a cooperative society as the number of people in the society increases. Now, we are concerned with how the dynamics in games (which cannot be described by static game models) affect the realization of cooperative society.

Our method of modeling evolution was described in Section 3. In most of the LD game simulations presented in this study, the number of trees was set to one. In this case, each player's available action is 'waiting' or 'cutting the tree'. We have studied both two-person and three-person DS games using both the linear and the convex maps for the natural law of the tree growth. (Thus, we consider four cases.) We focus on how the *number of persons* and the *different characteristics of the game dynamics* affect the evolution of the society. An evolutionary simulation was conducted three times for each of the four cases. We now discuss typical simulation results for each.

Fig. 2 shows how the fitness value changes as the generation of each simulation increases (fitness chart). (The fitness of the fittest species in each generation is called *the fitness value* (*of the generation*) here.)

In the *two-person linear LD game* (Fig. 2(a1)), a high fitness value is realized at an early stage and it fluctuates around 0.5 for some time. However, as the strategy evolves, some lumberjacks begin choosing to betray. From about the 500th generation, they start competing to cut trees, lowering the fitness value of the generation. Then, for a long time the society is ruled by betrayers. At about the 7500th generation, however, the lumberjacks form rules for cutting trees, a sign of cooperation in the society. Finally, the fitness value of the generation is stabilized around 0.5. Contrastingly, in the *three-person linear LD game* (Fig. 2(a2)), from approximately the 30th generation, the lumberjacks enter a competitive society, and they cannot get out of the mode of tree-cutting competition. (Though the number of trees on a hill is set to 1 here, we have found that a society of tree-cutting competition also results in three-person linear LD games when the number of trees is increased from 1 to 3.) This suggests that a cooperative society cannot be achieved as the number of persons increases in the linear LD game.

Next, let us consider the fitness chart of *two-person convex LD games* (Fig. 2(b1)). Here again, at an early stage, the fitness value of the generation drops suddenly. Then, after some fluctuations and jumps, it remains around a certain high value. In short, the society is able to emerge from tree-cutting competition, and it is transformed into a cooperative society at a relatively early stage. In this way, it is able to regain a high fitness value. Then, following several transitions in the fitness value, a cooperative society is established from about the 4500th generation with the fitness value fluctuating between 0.6 and 0.8. The average of the size of the tree is approximately 0.25

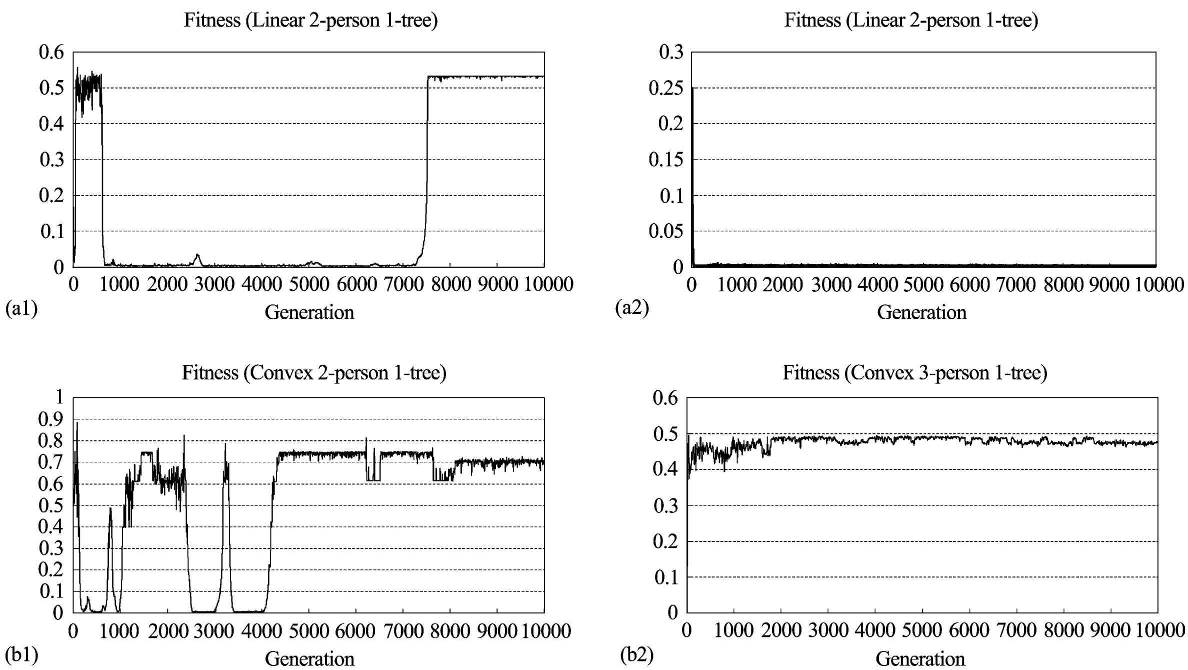

Fig. 2. Fitness chart: *fitness value* of each generation is plotted, with the horizontal axis representing the generations. The *fitness value* of a generation is defined as that of the fittest species in that generation. The fitness chart for a (a1) two-person linear LD game, (a2) three-person linear LD game, (b1) two-person convex LD game and (b2) three-person convex LD game are shown.

in later generations. It is thus seen that eventually a fertile game environment is created and maintained.

In the *three-person convex LD game* (Fig. 2(b2)), a cooperative society is formed at an early stage. Once this society is formed, the society never returns to a competitive state. The maximum fitness value fluctuates near 0.5. The approximate mean value of the tree height is 0.35, and a more fertile game environment than that in the two-person game is maintained. (Of the three simulations for the three-person convex LD game, one turned into a lawless society at an early stage, but a cooperative society was established at about 6000th generation and maintained steadily after that, see Appendix B.)

The important conclusion of this section is the following. An increase of the number of players works as an obstacle to the creation and maintenance of a cooperative society in the linear LD game, as the static game model (such as the n-person Prisoners' Dilemma). In the convex game, however, an increase of the number of persons does not prevent the creation of a cooperative society. On the contrary, in this case it appears that a more cooperative society can be established, as evidenced by the results for the three-person game.

Note that when considered in the framework of a static game, the linear and convex LD games have identical social dilemmas. The only difference lies in the rule of the dynamics of the tree growth. Hence, in order to understand why a cooperative three-person society can be established and well maintained in the convex case, we need to determine how the dynamic structure of the game depends on the nature of the tree growth by examining the simulations in detail and comparing this structure in the linear and convex cases. In the next section, we first present characteristic phenomena for the two-person, one-tree convex LD game. In Sections 6 and 7, we elucidate the mechanism underlying in these phenomena, focusing on the dynamics of the game. There, the formation and maintenance of cooperation and the increase in number of betrayals are studied in terms of DS. In Section 8, we discuss the evolutionary LD games under different conditions (number of players, number of trees, and the rule for tree growth). Finally, in Section 9, we discuss the social dilemma and DS games in general by drawing together the results of these investigations of the LD games considered in this study.

5. Simulation of the two-person, one-tree convex LD game

5.1. Outline of the evolutionary process

In this section, we will discuss specific phenomena seen in the simulations of the two-person, one-tree convex LD game.

First, we give an overview of the simulation results (Fig. 2(b1)). In early generations (roughly up to the 1000th), lumberjacks attempt to compare with each other by cutting as many trees as they can. As a result, cooperation does not appear in the society; that is, players are likely to betray each other on almost all the hills. However, as generations pass, lumberjacks begin making rules of cooperation. These rules of cooperation adopted in the society gradually change as generations go by, and as a result, the state of cooperation sometimes collapses completely and is replaced by a competitive society. In this way, cooperative societies form, change, and collapse repeatedly over generations (until approximately the 4000th generation). Eventually, however, a cooperative state is established completely, and a non-cooperative society no longer appears.

We now examine the results of the simulations, and in the process clarify the origin of a cooperative society.

5.2. Bias toward betrayal

Since the LD game possesses the social dilemma, there always exists a structural bias toward betrayal, as in the Prisoners' Dilemma. For this reason, at early generations, selfish action with exploitation is common among players.

Fig. 3 displays the fitness chart for the early period (up to the 1000th generation). Initially, it is seen that lumberjacks' strategies are distributed randomly. Here the fitness value exhibits large fluctuations, since

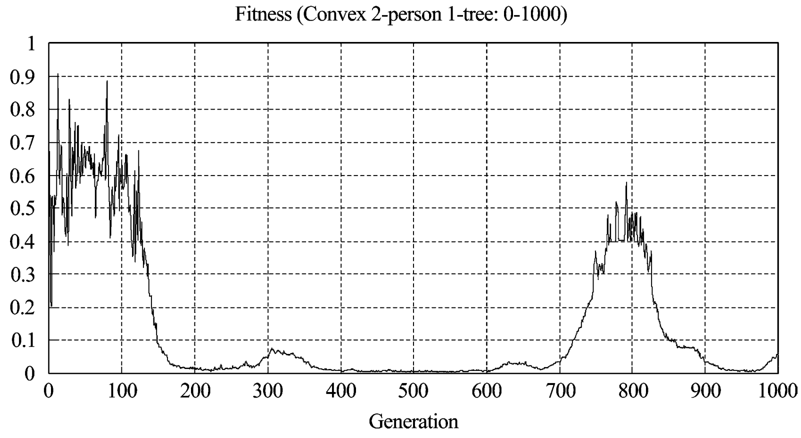

Fig. 3. Fitness chart of early generations for the two-person and one-tree convex LD game simulation.

players undertake trial-and-error behavior to test a variety of sequences of actions (cutting the tree and waiting for tree growth) almost randomly. The fitness value of the generation found in this period is sometimes high. This is not because the strategies have evolved enough for lumberjacks to form cooperation, but because the above randomness sometimes lowers the frequency of tree-cutting action and makes a productive game environment accidentally. Then, as evolution progresses the fitness value drops suddenly around the 100th generation when players start to compete in cutting trees. Such competition and its effect on the game environment can be observed in the *game dynamics* on the hill: (1) the dynamics of the players' action; (2) the dynamics of the tree size; (3) the dynamics of the players' state.

The *action chart* (Fig. 4) shows how the players in the LD games are relentless in cutting trees. The frequency of cutting trees becomes higher after this competitive behavior begins. Accordingly, as seen in the *resource chart* and *state chart* (Fig. 5(a) and (b)), the height of the tree and the state of the player remain at low values.

In any single round of this game, the benefit of cutting the tree always exceeds the payoff of waiting. In this sense, the above described blind competition for cutting trees during this period is a natural consequence in a game characterized by the *Tragedy of the Commons*. In fact, selfish strategies with non-cooperative behavior are commonly observed in many kinds of LD games, and they are not limited to the present two-person convex LD game. Increasing one's individual benefit through selfish behavior is an easier tactic for evolution than increasing the overall benefit by cooperating with other lumberjacks, which requires strategies to coordinate behaviors with others.

In order for cooperative behavior to spread among players, some agreement on the definition of 'cooperation' needs to be made among them. However, in the present LD game, no specific symbols representing

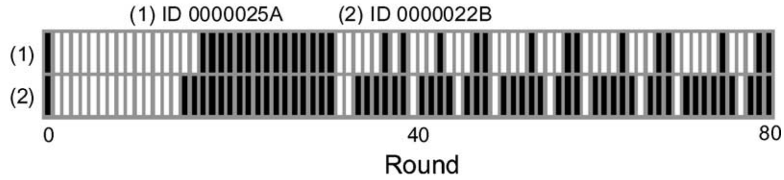

Fig. 4. Action chart (200th generation): a black rectangle indicates the action of 'waiting', and a white rectangle that of 'cutting the tree'. The horizontal axis represents the round. From the total of 400 rounds, the first 80 rounds are shown. 'ID 0000025A' and 'ID 0000022B' are the names of the two species to which the two lumberjacks belong.

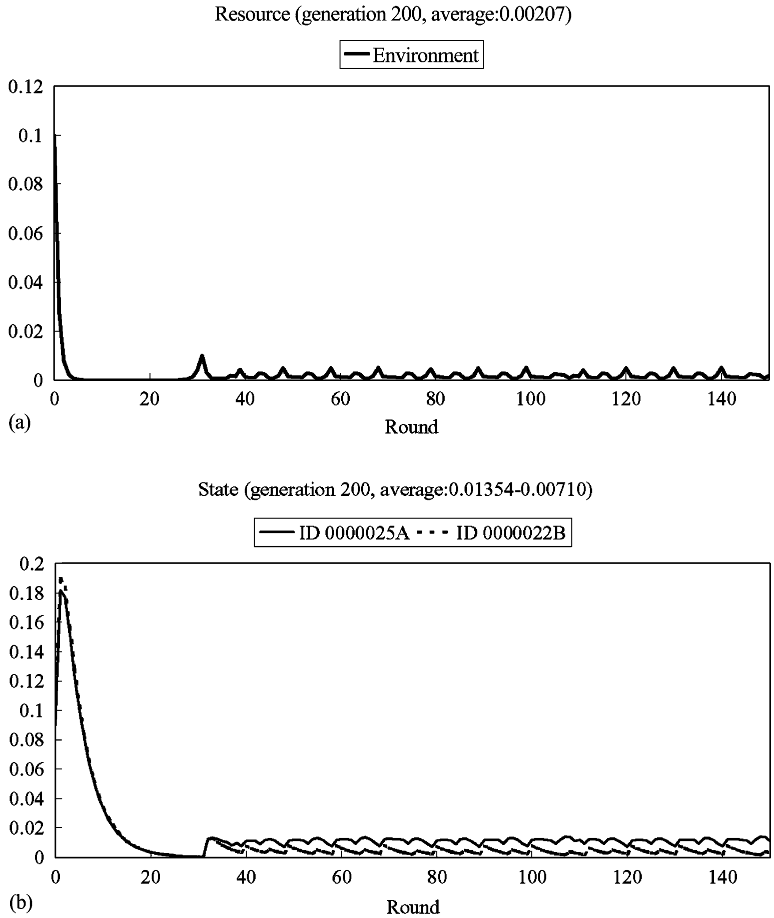

Fig. 5. (a) Resource chart: the height of the tree is plotted as a function of the round number. (b) State chart: the state of the player is plotted as a function of the round number. Both charts are corresponding to the action chart, Fig. 4. Throughout this paper, for the first round of the game, the tree height is set to 0.1, and the state of each player is chosen from random numbers from the normal distribution with the mean 0.1 and the variance 0.1 (see Section 3.2.2).

"cooperation" or "betrayal" are given in advance as a rule. Furthermore, since variables (such as the size of the tree) are continuous, the value of benefits reaped has innumerable possibilities, depending on the timing when a tree is cut. Thus, social standards regarding cooperation, such as the minimum tree height necessary for cutting, are not specified in the construction of the model, in contrast with the traditional games, such as the Prisoners' Dilemma. Such standards must be formed among the players through evolutionary dynamics. For this reason, more is demanded in this LD game with regard to achieving cooperation in comparison with the traditional games.

In spite of the above difficulty a cooperative society is formed. First, it is temporarily created around the 800th generation, resulting in an increased fitness value. This cooperative state, however, soon collapses at a little later generations, being replaced again by a competitive society, which lasts a long time. In this simulation, the large-scale collapse of the cooperative society occurred four times (Fig. 2). In the present simulations, the evolution toward a competitive society was found to accelerate, once the fitness value of the generation fell below 0.4. The cause for this collapse of a cooperative society is discussed in later sections.

5.3. *The formation and transition of cooperation rules*

There are many possible types of norms regarding the cooperation agreed among the players in our LD game. For example, behavior in which an individual only cuts a tree if it is taller than 0.7 might be called cooperative in one society, while it might be deemed selfish in another. Through the evolutionary process, a society keeps changing its norm for cooperation. In our simulation, first, a certain type of cooperative society was formed and maintained for a short time. Then, it was replaced by a different type of cooperative society. These process was repeated again and again. Now, we study the changes in the cooperative norms observed in our game-world.

5.3.1. *Stepwise evolution (from 900th to 1800th generation)*

Fig. 6 (a blow-up of Fig. 2(b1)) displays fitness charts from the 900th to the 1800th generation. As can be seen, from about the 1000th generation, the fitness value of the generation (referred to as the "fitness value") begins to rise step-by-step. Gradually, in a stepwise fashion, a cooperative society is created by moving away from the competitive state. For each epoch that displays such stairs, a type of game dynamics specific to that epoch dominates the game-world, i.e. the 60 hills used in the simulation. By contrast, epochs without such stairs exhibit many different kinds of dynamics distributed over the various hills. In the former case, between each step, the society changes drastically. Let us take a close look at three epochs: Epochs A–C (see Fig. 6).

Let us name the distinct game dynamics characterizing each of the Epochs A–C as types A–C. The dynamics are completely identical for all hills in some cases, while they are nearly identical in the other cases. For instance, type-A game dynamics exist on more than half of the 60 hills during a certain generation of Epoch A, while during all other generations of this epoch, type-A dynamics exist on almost all 60 hills.

5.3.2. *Characteristic dynamics in Epoch A–C*

Fig. 7 displays type-A game dynamics that are dominant in *Epoch A*. As indicated in Fig. 7(a) and (a)′, the players here exhibit the period-5 action sequence of "wait, cut, wait, cut, wait". In addition, the two players' actions are identical. From the dynamics of the tree sizes (Fig. 7(b)), it can be seen that the lumberjacks collect lumber while allowing the trees to grow to some extent. The actions forming type-A are considered as the norm for cooperation for lumberjacks living in the game-world. During this epoch, the nature of the game is such that the mean height of the tree is approximately 0.12.

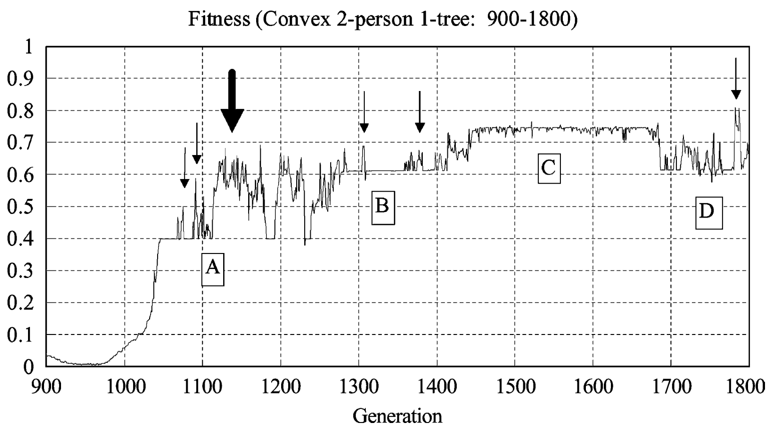

Fig. 6. Two-person, one-tree LD game: the fitness charts from the 900th to the 1800th generation (blow-up of the graph in Fig. 2(b1)). A–D in this figure denote Epoch A (1100th generation), Epoch B (1250th generation), Epoch C (1450th generation), and Epoch D (1700th generation).

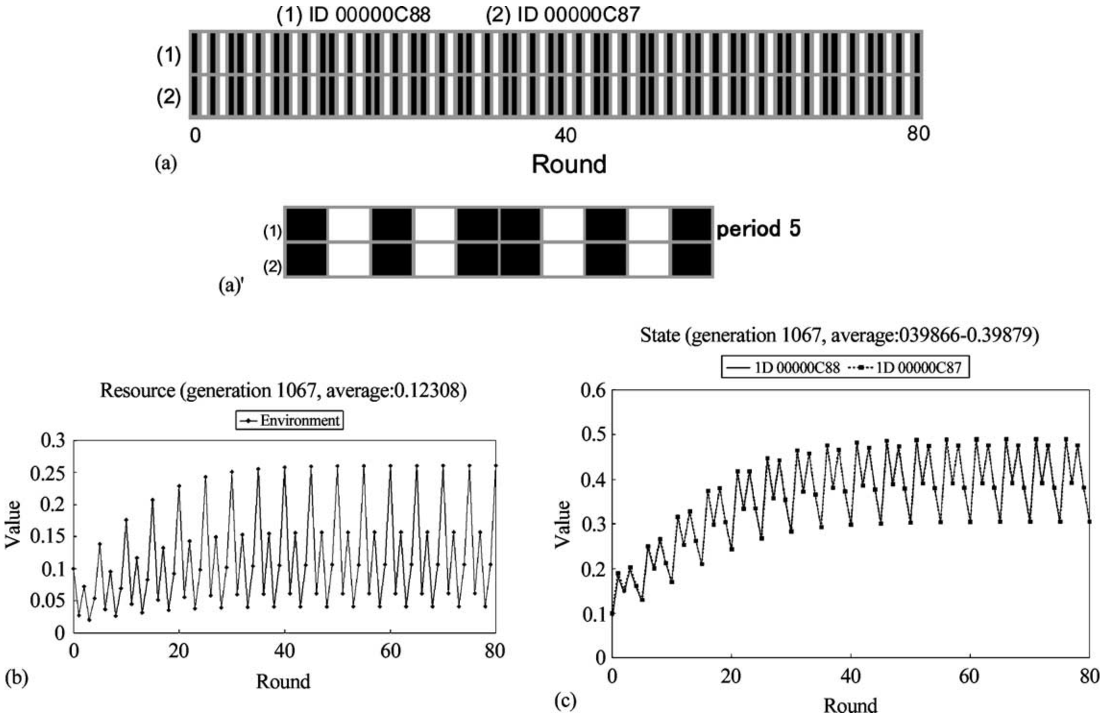

Fig. 7. Epoch A: (a) action chart (with (a)′ its attractor part); (b) resource chart; (c) state chart.

Fig. 8 displays the dominant game dynamics in *Epoch B*. During this epoch, the mean height of the tree is again approximately 0.27. The dominant action of the players in this epoch are given by the period-3 sequence "wait, cut, wait", as shown in Fig. 8(a) and (a)′. The frequency of cutting in this epoch is lower than that in Epoch A, and thus a more productive environment is maintained. The actions of the two players are also identical in these type-B dynamics. Dynamics similar to these are also seen in Epoch D (Fig. 9). The difference between Epochs B and D is only the proportion of the players that adopt this dominant action sequence. In Epoch D, the same dynamics are likely to be exhibited on all of the hills.

The nature of the game in *Epoch C* is such that the average tree height is approximately 0.45. As seen in Fig. 10(a) and (a)′ each player carries out the period-4 action sequence, "wait, wait, wait, cut". The most salient feature of this type of dynamics is that the two players are not synchronized in their action. Here the action sequence is performed out of phase by the two players and they alternately raise trees and gather lumber. As a result, a more productive game environment than that of Epoch A or B is created.

5.3.3. *Transitions between epochs*

It is clear that different types of dynamics are dominant in Epochs A–C individually and that each type is maintained over a certain time. As time passes from Epoch A–B to C, productive game dynamics are gradually generated. Of course, in some cases, productivity drops, as from Epoch C to D. We wish to know how the transition between two ages proceeds.

Judging from Fig. 6, the fitness within a given epoch is not completely constant. There are several pulse-like periods here and there (as indicated by the arrows). Such periods are common in Epoch A and some of them last too long to be called 'pulses'. During the time of a wide pulse around the 1137th generation within Epoch A (indicated by the wide arrow in Fig. 6), the fitness value is slightly larger than the mean fitness value of this epoch. In the generations during

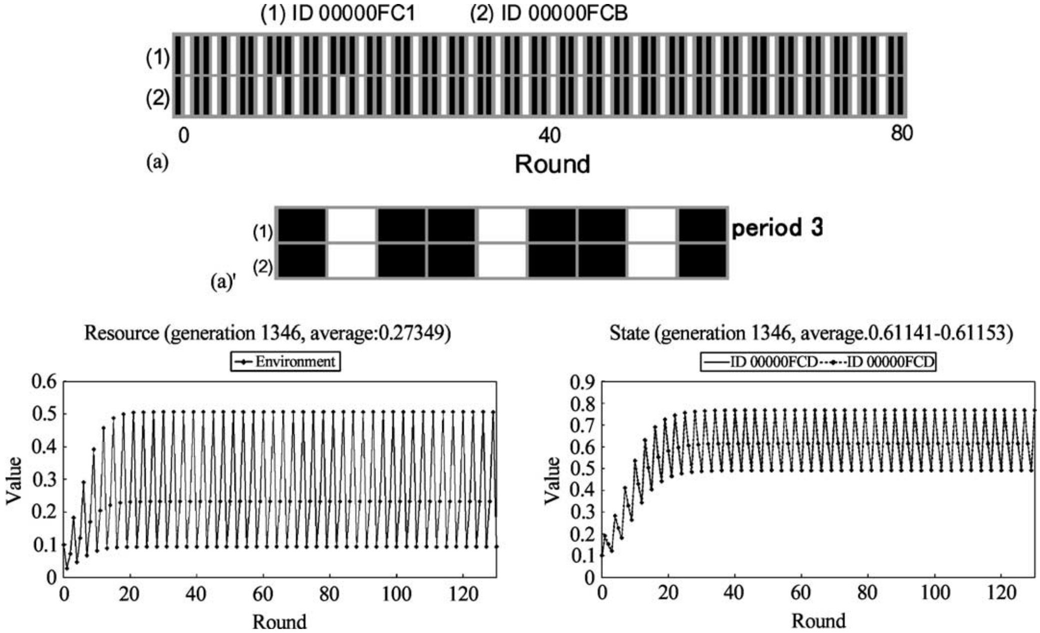

Fig. 8. Epoch B: (a) action chart (with (a)′ its attractor part); (b) resource chart; (c) state chart.

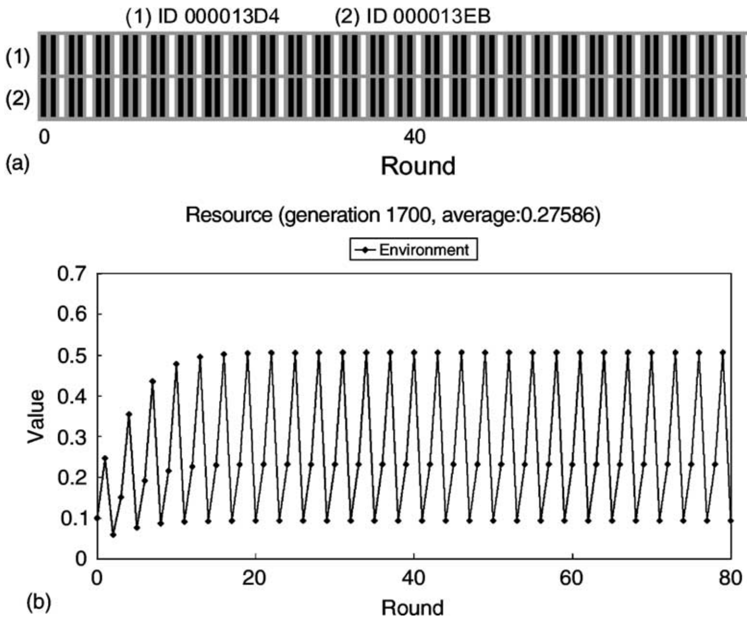

Fig. 9. Epoch D: (a) action chart; (b) resource chart.

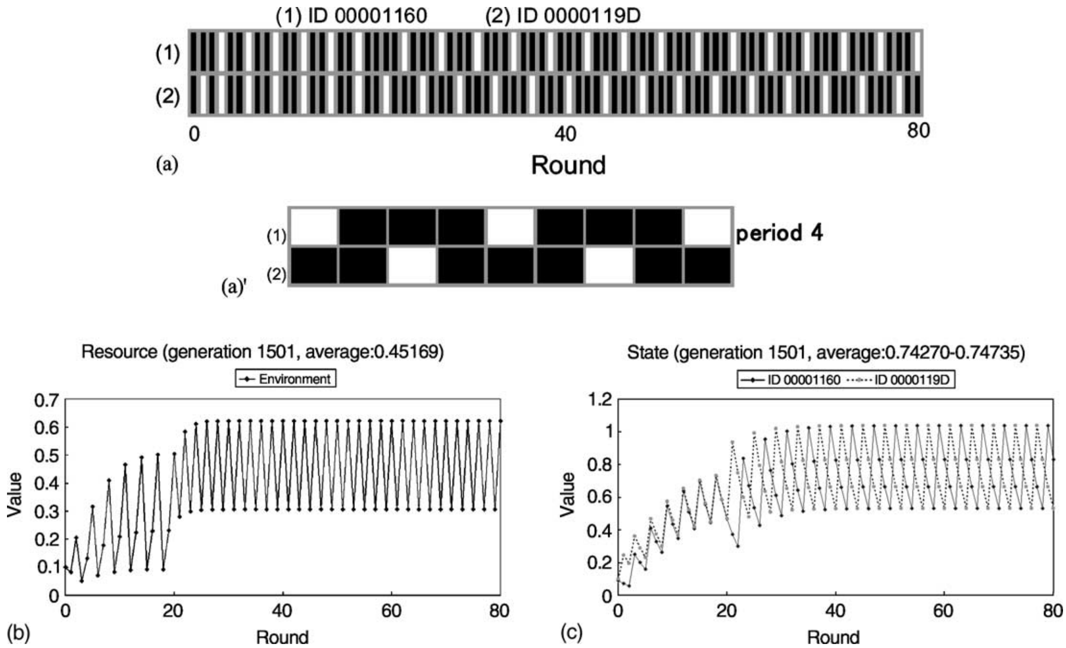

Fig. 10. Epoch C: (a) action chart (with (a)′ its attractor part); (b) resource chart; (c) state chart. In this case, beginning at the 22 round, each player waits for the other player to cut the tree. In this way, there comes to be a temporal differentiation of roles, and the actions fall into cyclic dynamics of period 4.

this pulse, dynamics of both types A and B are observed frequently. The game dynamics exhibited on an individual hill depend on the species (i.e. the strategy) of the lumberjacks on that hill. In the 1137th generation, some species can execute only type-A action sequences, while others can execute both those of types A and B, as in Fig. 11(a) and (b). There are a number of different kinds of dynamics displayed on the 60 hills in a generation of Epoch A. The numbers of lumberjacks displaying type-A dynamics, type-B dynamics and other types change as generations pass, but in general, the number displaying type-A is largest throughout

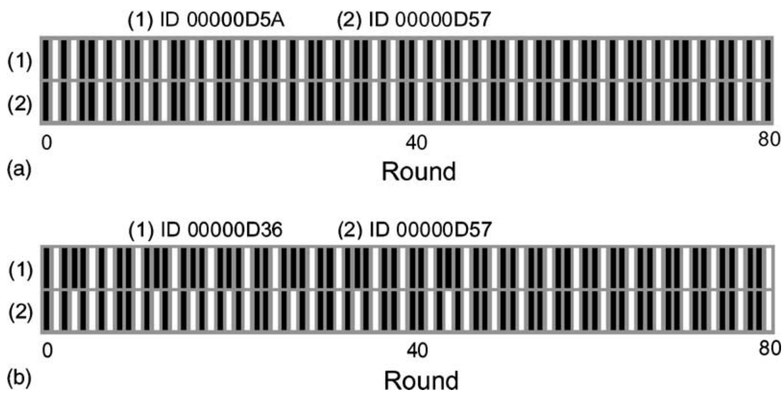

Fig. 11. Two types of game dynamics observed at the 1137th generation (Epoch A): (a) and (b) action diagrams of type-A and type-B dynamics, respectively. In both (a) and (b), there is one player from species ID-00000D57. Depending on the opposing player, this player employs either type-A or type-B dynamics.

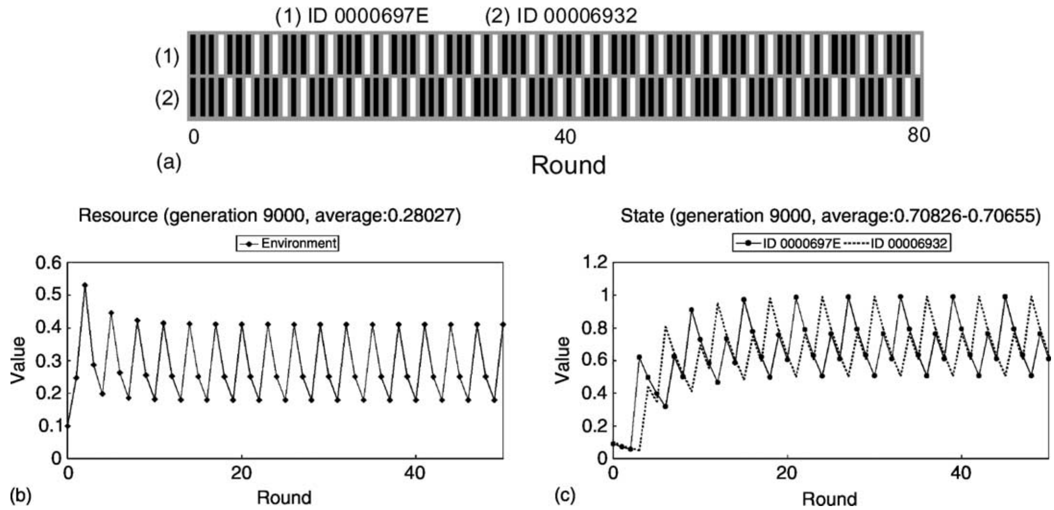

Fig. 12. 9000th generation: (a) action chart; (b) resource chart; (c) state chart.

the epoch. In Epoch A, there are occasionally switches between types A and B. For instance, the number of lumberjacks displaying type B is larger at the 1137th generation, mentioned above. Still, as long as the transition to type B is not complete, the society eventually returns to type A. The pulse phenomena in Epoch A exemplify this kind of switching process. Conversely, Epoch B is characterized by the stabilized state that appears when the transition to type B is complete.

5.3.4. *Later generations*

Later in the evolutionary process (Fig. 2(b1)), after the 2000th generation, the society continues to change the rules of cooperation, and the fitness values also continue to change. Although, during this time, occasionally a cooperative society collapses and turns into a competitive society (at about the 3000th and the 4000th generations), a cooperative society is more easily recovered as generations proceed. After the 4000th generation, the society never again becomes competitive, and a cooperative society is established, although the norm for “cooperation” continues to change. Fig. 12 illustrates the dominant game dynamics of the 9000th generation. Here, two lumberjacks manage to grow the tree by having time-sharing for cutting trees with a different phase, and changing the role in turn, as discussed above in the case of Epoch C.

6. Stabilization of a cooperative society and changes of rules

6.1. *Stability of the game dynamics—analysis using AGS diagrams*

In the previous section, we observed that lumberjacks form rules to manage the dynamics of the resources cooperatively in the two-person, one-tree convex LD game. Specifically, they grow and consume the natural resources together or alternately, periodically changing their action. Also, the rules for cooperation change with generations. How can such formation and development of the rules be possible? How can such forms of cooperation remain stable given the social dilemma that exists? To answer these questions, we will analyze the relationship between the evolution of strategies and the game dynamics using the *AGS diagram*, which was introduced in a previous paper [1], to show the change of attractor of game dynamics due to the change of strategies. (See Appendix A for a brief introduction to the AGS diagram, which is necessary for the latter part of this paper.)

Note that (a) *decision making of the players* determines the (b) *game dynamics* and therefore determine the (c) *average scores of the players*. AGS diagrams can elucidate the relationship between (a) and (b), while the 'average-score landscape', used in Section 6.3, reflects the relationship between (a) and (c). Here, using AGS diagrams, we mainly discuss the stability of the game dynamics in a cooperative society.

6.1.1. Articulated structure in the AGS diagram

First, let us consider Epochs C and D (Fig. 6). Their dominant game dynamics are displayed in Fig. 10 (1501st generation) and Fig. 9 (1700th generation). The decision-making functions (Section 3.2.3) of the fittest species of these epochs are shown in Fig. 13(a) and (b). We call the fittest species of the 1501st generation 'Species C1' (and a player of this species 'Lumberjack C1'), and that of the 1700th generation 'Species D1'. The characteristic game dynamics exhibited in Epochs C and D are completely different. Nevertheless, the decision-making functions in these epochs are quite similar. The main differences are in the values of θ_{11} and θ_{20}. For example, θ_{11} is approximately -0.65 for Species C1 and -0.15 for Species D1.

Next, let us study the AGS diagrams. The LD game played by a player of Species C1, which is the fittest species of the 1501st generation, and a player of the second-fittest species results in the period-4 dynamics displayed in Fig. 10. The AGS diagrams for the game played by two such lumberjacks appear in Fig. 14, with the value of θ_{11} appearing there being that of Lumberjack C1.

As seen in the figure, near the parameter value -0.5, there is a period-3 attractor, as indicated by the three parallel segments existing there. These segments correspond to dynamics of type D. On the other hand, the two dark segments ranging from about -0.4 to $+1.0$ correspond to the period-4 action sequence of type C dynamics.[3] Let us call such a parallel segment area a *plateau*. The value θ_{11} of Lumberjack C1 (about -0.65) is within the period-4 plateau. With a change of θ_{11} to the period-3 plateau (about -0.15) the player comes to exhibit type-D dynamics. Difference between Epochs C and D is produced by such a difference in strategies, and is due to the dynamical structure of this two-person convex LD game.

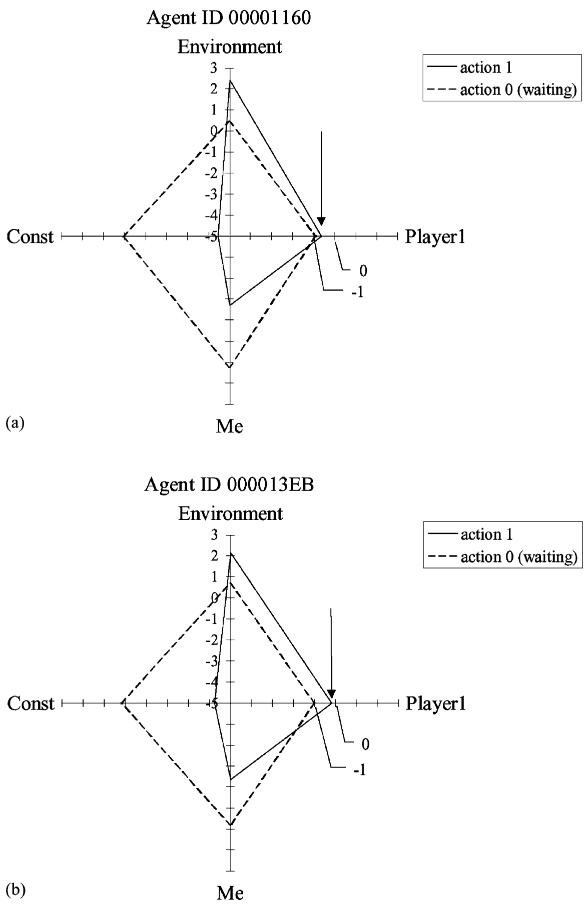

Fig. 13. (a) and (b) Decision-making functions of the fittest species of the 1501st generation (in Epoch C) and the 1700th generation (in Epoch D), respectively. The solid line and the dashed line indicate the incentive for action 1 (cutting the tree) and action 0 (waiting), respectively. Here, the points of intersection between the solid line and the 'Environment' axis, the 'Player 1' axis, the 'Me' axis, and the 'Const' axis indicate the values of the corresponding coefficients of the decision-making functions, η_{11}, θ_{11}, θ_{21}, and ξ_1, respectively, while the points of intersection between the dashed line and these axes give η_{10}, θ_{10}, θ_{20}, and ξ_0, respectively. Although the game dynamics are all significantly different, as we can see from (a) and (b), the decision-making functions are quite similar. The slight difference in the value of θ_{11} that causes the significant difference in dynamics is indicated by the arrow.

When a stable cooperative society in the two-person convex LD game is realized, as seen in Fig. 14,

[3] Although the players' actions are attracted to the period-4 action sequence, the dynamics of the tree height are period 2. This is because the dynamics of the tree size depend solely on whether it is 'cut' or 'not cut', independent of the player who cuts the tree.

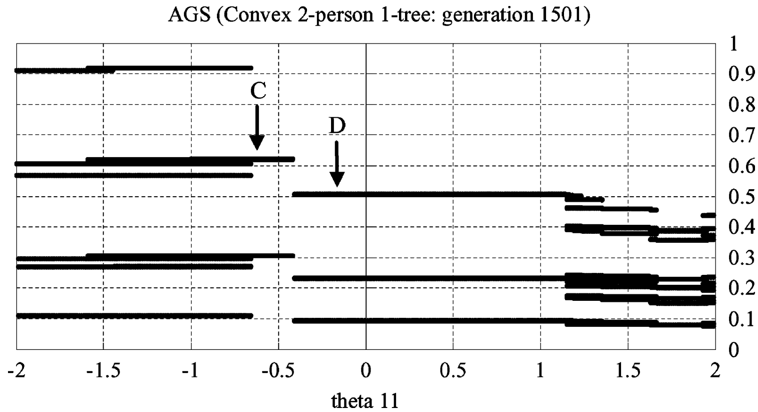

Fig. 14. AGS diagram: attractors of the dynamics of the tree size are plotted for a range of values of θ_{11} for two lumberjacks in the 1501st generation. The two segments indicated by the arrow C represent a period-2 attractor of the tree height type-C dynamics. The three segments indicated by the arrow D represent a period-3 attractor consisting type-D dynamics.

almost all of the AGS diagrams of the lumberjacks are composed of plateaus of periodic attractors.[4] Within the plateau, the game dynamics do not change even when there is a change in strategy. For example, in Fig. 14, the θ_{11} value of Lumberjack C1 is approximately -0.65, within the plateau of type C. However, even if the θ_{11} value of this player has a slight deviation due to mutation, the game dynamics to be observed will remain the same. If θ_{11} exceeds -0.4, the game dynamics will jump into type D immediately. Afterwards, even if θ_{11} is further increased, the game dynamics will remain type D.

From the viewpoint of lumberjacks' strategies, the structure of game dynamics is clearly articulated with many plateaus in the AGSs of the lumberjacks when the mutual cooperation is formed among them.

6.1.2. Plateaus in AGS diagrams and their effect on the evolutionary phenomena

In our LD game simulations, several cooperative societies, each of which exhibited distinctive game dynamics, succeeded in lasting over several generations.

These game dynamics are also found among the plateaus in the players' AGS diagrams. We refer to these dynamics as *strategic metastable game dynamics* (SMD). In SMD, lumberjacks live in a cooperative society that exhibits game dynamics that are stable with respect to *changes in some players' decision-making functions*. In other words, some plateaus in the AGS diagrams correspond to SMD that can create cooperative societies.[5]

When the system displays cooperative behavior, the game dynamics do not change gradually as the strategy changes. Rather, they suddenly jump to different game dynamics at certain critical points. In the cases of Epochs A–D, certain kinds of game dynamics are dominant in the game-world. These all correspond to plateaus in the AGS diagrams of lumberjacks existing in those epochs. In other words, these plateaus define standards for the society or the rules that most players follow. Their very existence is a necessary condition to establish a stable cooperative society. (The *sufficient condition* is given in the following subsection.) We will refer to a plateau as an SMD if these sufficient conditions are satisfied. SMD expresses a mutual agreement among players acting cooperatively.

In the present study, the observed transitions between different cooperative societies always cor-

[4] We have already observed such plateaus in the AGS diagram in the one-person convex LD game [1]. However, the structure in the present case is completely different from that in a one-person game.

[5] According to this definition, treasonous game dynamics in a competitive society that persist for a long should be called SMD, but in this study, we restrict "SMD" only to that which forms a plateau for a stable cooperative society in the AGS.

respond to the transitions between different types of SMD. The stepwise development of cooperation rules follows the stepwise structure of the SMD in this game.

The LD game is played out in terms of continuous variables. Thus, there are potentially innumerable varieties of dynamics in action, resource, and states. However, for the 'cooperative society' seen in this two-person convex LD game, there are only a few types of game dynamics that we have observed. In fact, it is often the case that only one type of game dynamics is observed throughout the entire 60 hills in the game-world. Such a reduction of the game dynamics is reflected by the articulated structure in the AGS diagrams. As will be shown in the following subsection, the sufficient condition referred above for the stabilization is satisfied due to the limitation of the number of dynamics. As is shown in Section 7, if such articulated structure of the dynamics does not exist, the social dilemma of the LD game cannot be resolved, and a cooperative society can not be achieved.

Now let us study how cooperation is created and stabilized when the game dynamics are articulated.

6.2. Mechanism responsible for the stabilization and development of cooperative rules

In this subsection, we examine how cooperative rules are established and stabilized and how they develop into new rules.

6.2.1. Robustness of two-way strategies: transition from Epoch A to B

First, let us study how the transition from Epoch A to B takes place and how type-B dynamics are stabilized.

During the transitional period between Epochs A and B, two types of species are mainly observed in the game-world. We call the one 'Species A' (and a player of this species 'Lumberjack A'), and the other 'Species B' (and 'Lumberjack B', correspondingly). Through the selection process, the number of Species B eventually surpasses that of Species A (Fig. 6).

Table 3 gives the average scores for Lumberjacks A and B. In a game played by two lumberjacks of Species A, the game dynamics are type A, and the average score is 0.4 for both. (There is some fluctuation of the score for each game around this average, though.) Meanwhile, Lumberjack B employs type-A dynamics for the game with Lumberjack A, and as a result the average score is 0.4 for both. For the game with two lumberjacks of Species B, type-B dynamics are employed, and the average score is 0.6 for both lumberjacks. In other words, Lumberjack B is able to successfully use both type-A and type-B dynamics.

Table 3
Score table for Lumberjacks A and B (at the 1250th generation)[a]

Name	Score	Comparison	Score	Name
A	0.4	=	0.4	A
A	0.4	=	0.4	B
B	0.6	=	0.6	B

[a] The values are approximate average score for lumberjacks in an LD game of 400 rounds.

Due to the dominance of Species B over Species A during this transitional period, the former gradually increases its population in the game-world,[6] thereby establishing Epoch B. The evolutionary stability of Species B can be understood as follows. Suppose that $E(X, Y)$ is the score of Strategy X against Strategy Y. Then, the condition for Strategy I to be an *evolutionary stable strategy* (*ESS*) is that for all Strategies J ($I \neq J$), (1) $E(I, I) > E(J, I)$ or (2) $E(I, I) = E(J, I)$ and $E(I, J) > E(J, J)$ [16].

From Table 3, it is clear that the strategy of Lumberjack B satisfies condition (1). If we assume that most strategies in this period are limited to either Species A or B,[7] the increase in the Strategy B and the stability of Species-B society is thus explained. In other words, if the game dynamics are articulated, as discussed in the previous subsection, due to the characteristics of the dynamical law of this LD game, and all lumberjacks are in the state of employing dynamics

[6] This process is similar to that seen the evolution displayed in the imitation game [15], since B can imitate A, but A cannot imitate B.

[7] This is true in the case that, in the AGS diagram, lumberjacks' strategies are concentrated near the border between the two plateaus of types A and B.

Table 4
Score table of Lumberjacks C and D

Name	Score	Comparison	Score	Name
C	0.7	=	0.7	C
C	0.5	<	0.8	D
D	0.6	=	0.6	D

of either type A or B, then the transition from Epoch A to B can be explained by ESS. We can also explain the transition from Epoch B to C, in a similar manner.

6.2.2. *Continuous nature of the LD game and the level down of the game dynamics from Epoch C to D*

Next, let us take a look at the transition from Epoch C to D (Fig. 6). Since the fitness value drops at this transition, the previous explanation of the transition from Epoch A to B cannot be used here.

Table 4 gives the scores for two lumberjacks of Species C and D during the transition period from Epoch C to D (at 1685th generation). In the game involving two Species C lumberjacks, type-C game dynamics are used and the average score is approximately 0.7 for both lumberjacks. In the game involving two Species D lumberjacks, type-D game dynamics are used, and the average score is approximately 0.6 for both, which is smaller than the average score between C lumberjacks. However, in the game between Lumberjacks D and C, Lumberjack D exploits the Lumberjack C. His average score is 0.8, which is larger than the average score in the game with two Species C lumberjacks. We conclude that, if the distribution of strategies is concentrated around those of Lumberjacks C and D, the population of Lumberjack D will increase.

The reason for the success of the invasion by Lumberjack D lies in the *continuity* in the LD game. Among the strategies classified into type C, there is continuous set of slightly different values. Although the *type* of a species is determined by the corresponding *attractor of the game dynamics*, the parameters in the decision-making function for a given species can vary continuously over certain ranges. In the early stages of Epoch C, immediately after the end of Epoch B, the parameter values for Species C are selected so that it has the ability to successfully compete against Species B, which is less cooperative. Species C at this stage possesses a strategy 'strict' against the type-B (and type-D) dynamics that are more selfish. However, with the success of Species C, almost all lumberjacks begin to use type-C game dynamics as generations pass. At this stage, it is most advantageous for a lumberjack to adopt the more cooperative type-C game dynamics, as quickly as possible. For example, Fig. 10 shows that the type-C dynamics begin after 20 transient rounds. By decreasing the length of this transient period, the average score from the total 400 rounds is increased. As a result, as periods pass, there is a continuing trend toward an increase in the number of lumberjacks possessing strategies characterized by greater "generosity" among the range of type-C dynamics. After this trend in evolution reaches some point, the lumberjacks of Species C are now so generous that they can no longer successfully compete against lumberjacks of Species D, who cut the tree more frequently. This allows for the invasion of the relatively selfish strategy.[8]

6.2.3. *Summary of mechanisms allowing escape from the 'tragedy'*

Now let us summarize the formation and stabilization of and transitions among cooperative societies in the LD game, the DS game model that we consider as a model of the *social dilemma*:

(1) The key for establishing cooperation lies in the articulation of the game dynamics, as reflected by the AGS diagrams in Fig. 14. Whether or not articulation is possible depends on the dynamical law of the game-world. For some dynamical laws, it is difficult or impossible to realize such articulation. For example, in the linear LD game, which uses a piecewise linear map for the growth of trees, it is very difficult to realize articulation of the game dynamics (see Section 7). Furthermore, even in the convex LD game, articulated structure is rarely observed in a completely non-cooperative society. In cooperative societies of the convex LD game,

[8] A similar mechanism causing the collapse of cooperativity due to excessive generosity is also seen in the collapse of money [20].

some articulation is realized through the interaction of the lumberjacks, and a discrete structure is created in the originally continuous LD game.

(2) The cooperative norms of the society are formed as *stable game dynamics* that result from articulation. In spite of the existence of the social dilemma in the LD games, its detrimental effect is avoided as a result of the dynamical structure created within the game. There are several such stable types of dynamics that yield cooperation. These different types correspond to different epochs of the stable society, each adopting a different norm. Evolution from one stable society to another can be analyzed by studying the dynamics of the scores for the different strategies articulated from a continuous range of parameters.

(3) After a cooperative society is achieved as the result of the articulation of dynamics, it is sometimes taken over by a more selfish society. This is because the norm adopted for cooperation within this established society becomes too generous through the evolution of the continuous parameters, and the society becomes vulnerable to the invasion of selfish strategies.

6.3. Stability with respect to the invasion of unrealized strategies—investigation using the "average-score landscape"

To this point, we have studied the generation of the cooperative society observed in the simulation. However, innumerable types of strategies are possible, in addition to those that actually appeared in the simulation, since a continuously infinite number of decision-making functions can exist, described by DS with real parameter values. Now, we examine if the dominant strategies found in the cooperative society at a certain generation can successfully compete with several strategies that differ from those found in the simulations. In particular, we study if these strategies are stable with respect to invasion by certain selfish strategies.

As an illustration, we consider two types of cooperative strategies that appeared in the simulation. In Fig. 15, the fitness chart from the 4000th to 4500th generation is given. In the cases considered here, a cooperative society is gradually created, while competition in cutting trees is gradually replaced by cooperative behavior. Although the cooperative rules

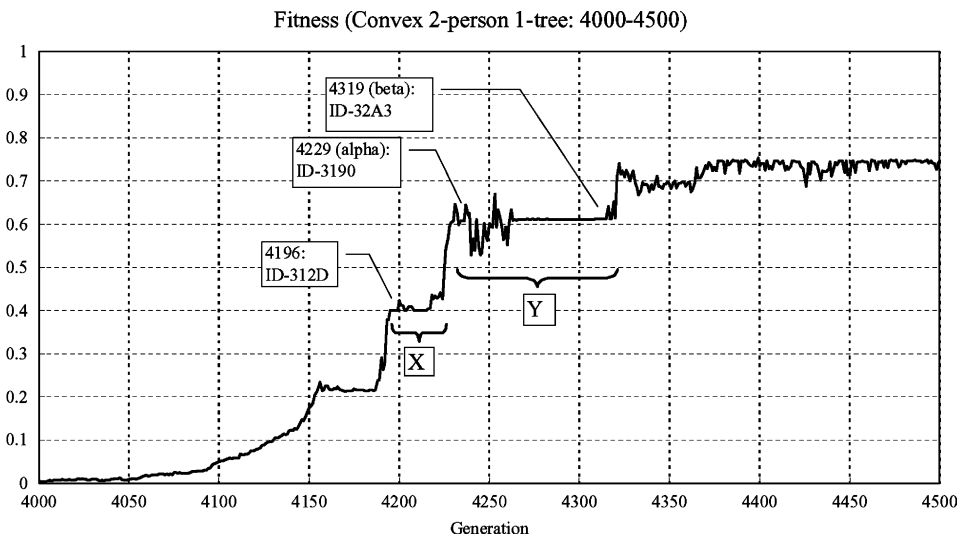

Fig. 15. Fitness chart from the 4000th to the 4500th generation. In Epoch X, a period-5 action sequence is dominant, while in Epoch Y, a period-3 action sequence is dominant. The 4196th generation is located in early Epoch X, while the 4229th and 4319th generations are located in early and late Epoch Y, respectively.

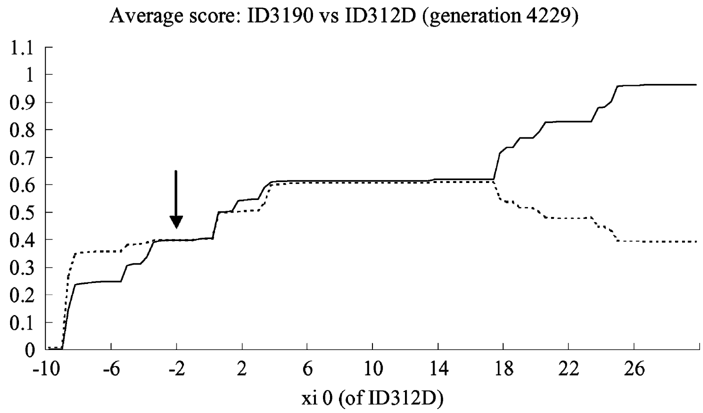

Fig. 16. The average-score landscape of α-player (1). The figure depicts the average scores of α-player and opponents. The opponents represented in this figure are players obtained as mutations of species 312D, the fittest species of the 4196th generation (in Epoch X). To be specific, the mutant is obtained by using all of the parameter values for species 312D, except that of ξ_0. This value is continuously changed from -10 to 28 for the mutant species. The value of ξ_0 for the opponent is indicated by the horizontal axis. The solid line represents the average score of α-player (400 rounds of the LD game), while the dotted line represents the average scores of the opponents. Note that the actual value of ξ_0 for the fittest species of Epoch X is about -2.3, as is indicated by the arrow.

occasionally change in later generations (after the 4500th generation), we have found that the cooperative society itself is *never* taken over by a selfish society.[9] Two phases, "Epoch X" and "Epoch Y", appear in the manner described above, as in Fig. 15. Here we focus our attention in particular on Epoch Y, in which a period-3 action sequence is dominant. Let us call the 4229th generation (early in Epoch Y) the "α-generation", and the 4319th generation (late in Epoch Y) the "β-generation".

6.3.1. Stability of the α-player respect to various competing strategies

First, we consider *a player* with the optimal strategy of the α-generation (strategy ID: 3190), which we call the "α-player" hereafter. We investigate here the performance of the α-player when he competes against players of various other strategies, each of which is a mutant of the optimal strategy of the 4196th generation in Epoch X. More precisely, the mutants are created by changing the parameter ξ_0 of the decision-making function of the optimal strategy of the 4196th generation. Fig. 16 displays the average scores of the two players in all the games. We call this type of graphs an *average-score landscape*. As we can see, the average score of the two players never exceeds 0.6. Note that when two of the α-players (or β-players) participate in a game, they employ the period-3 game dynamics that are characteristic of Epoch Y, and the average score is approximately 0.6 for both players. The results obtained here provide evidence that no strategy introduced here can overthrow the society that the α-players control [16].

Next, let us study the immunity of the society against a selfish player. As such selfish players, we prepare mutants of the optimal strategy of the 200th generation, which is a completely non-cooperative society. Fig. 17 displays the average scores of the games played between α-players and those selfish invaders. As we can see here also, the average score of the invading players never exceeds 0.6. In other words, any strategy prepared here cannot overthrow the society of the α-players. Based on this result, we can say that the α-type strategy is also stable with respect to strategic invasions, which were not actually seen in simulations. Furthermore, the α-player remains memory of the battles carried out in a competitive age that his ancestors experienced.

[9] Simulations were conducted up to the 20,000th generation.

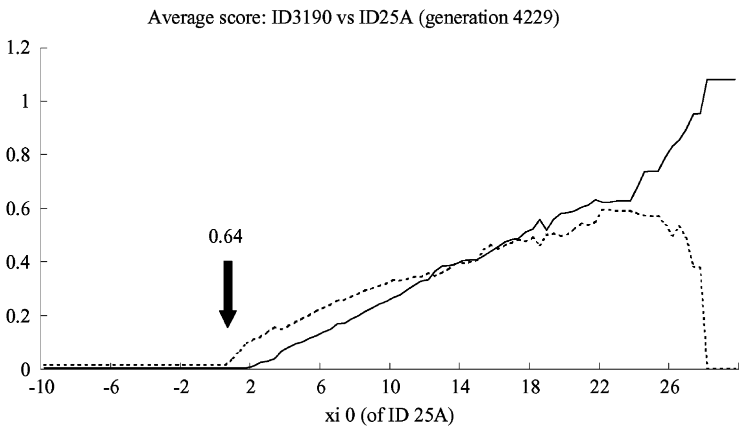

Fig. 17. The average-score landscape of α-player (2). The figure also depicts the average scores of α-player and opponents. The opponents represented in this figure are obtained as mutations of species 25A, the fittest species of the 200th generation. To be specific, the all parameter values of species 25A are used for the mutant, except that of ξ_0, which is continuously changed from −10 to +30. The actual value of ξ_0 for the species 25A player is about 0.64, as indicated by the arrow.

6.3.2. *Stability of β-player with respect to exploitative-type strategies*

Let us next examine the stability of the cooperative society of the β-generation with respect to selfish strategies. For this purpose, we pit a β-player against players of various selfish strategies. As the opponents, we again prepare mutants of the optimal (non-cooperative) strategy of the 200th generation.

Fig. 18 plots the average-score landscape for such confrontations. Here, it is clear that the β-player society is certainly stable with respect to the invasion of the (actual) optimal strategy of the 200th generation with $\xi_0 \approx 0.64$ (denoted by the narrow arrow), because the average score of the invader is far below 0.6. However, the average score of the invading players exceeds 0.6 when the parameter ξ takes a value near the wide arrows. That is, the β-player society would collapse due to the appearance of players with strategies obtained from the mutation of the ξ_0 value of the optimal strategy of generation 200 if these mutated values reach the value indicated by the wide arrow.

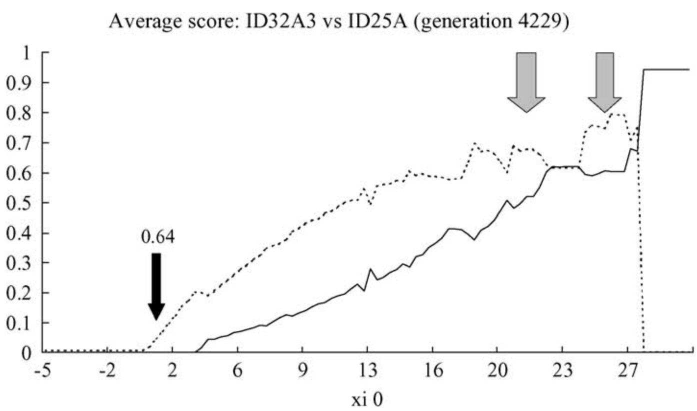

Fig. 18. The figure depicts the average scores of β-player and opponents. The opponents represented in this figure are also obtained as mutations from the fittest species (species: 25A) in the 200th generation. The actual value of ξ_0 for the species 25A player is about 0.64, as indicated by the arrow. We can see from this figure that the mutant strategies for which the value of ξ_0 is near wide arrows can successfully invade the society of β-players.

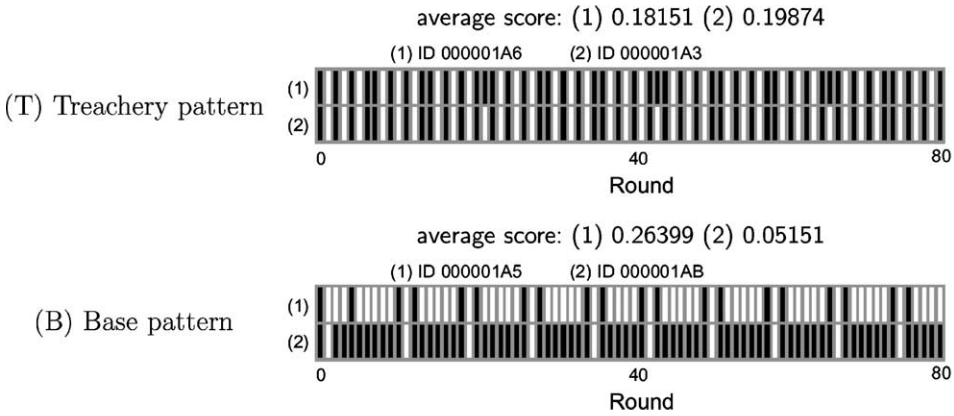

Fig. 19. Action charts at early times (140th generation). (B) The basic action sequence in this generation. Similar sequences are observed on many hills in the game-world of this generation. (T) One of the lumberjacks (species ID: 000001A5) acts selfishly, with a higher frequency of tree cutting. The average scores of the two players over 400 rounds also appear for each case.

Statistically speaking, such large mutations are almost improbable, since there is a very large difference between the parameter values of the strategies for the β-players and that indicated by the wide arrow. This is why the cooperative β-player society does not collapse in simulations and the game-world remains cooperative for a long time (at least up to the 20,000th generation).[10]

From the above analyses, we can conclude that the β-player society is not as stable with respect to various strategies as the α-player society. This is because the 'generosity' of the dominant strategy in the β-player society developed in Epoch Y, which exists a long time after the last battle against selfish players. In the actual simulations, however, this very generous nature of the β-players is advantageous, because, by being more generous, the lumberjacks can more effectively cooperate with others that possess various strategies (Fig. 15).

7. Decline of cooperation due to the continuous change of dynamics

In the example simulation we have discussed in this paper, the re-emergence of selfish behavior occurred four times. The collapse of a cooperative society and the increase of selfishness are often observed in the present model. We now analyze this kind of evolution toward selfish behavior by considering the structure of the game dynamics. We will also discuss the reason that an increase in the number of players results in a non-cooperative society for the *linear* LD game, but not for the convex LD game.

7.1. Non-cooperative societies in the two-person convex LD game

To understand the evolution toward selfish behavior, we plotted the action chart of a particular confrontation in Fig. 19. This shows the lumberjacks' actions on two hills of the 140th generation of the two-person convex LD game, when selfish actions are dominant.

Fig. 19(B) is the action chart of a battle that is somewhat characteristic of the game dynamics of the 140th generation, although there is no single typical battle here, since the game dynamics exhibited on the total 60 hills during this generation are highly diverse. This is a common situation for a selfish society in the convex LD game. In the battle represented in this figure, the two lumberjacks simultaneously cut trees about once every other time step.[11]

[10] Of course, the probability of collapse is not zero theoretically, however small it is, since the mutations are chosen randomly from a normal distribution (whose tails extend to ±∞).

[11] More precisely, the game dynamics observed on this hill are period 22.

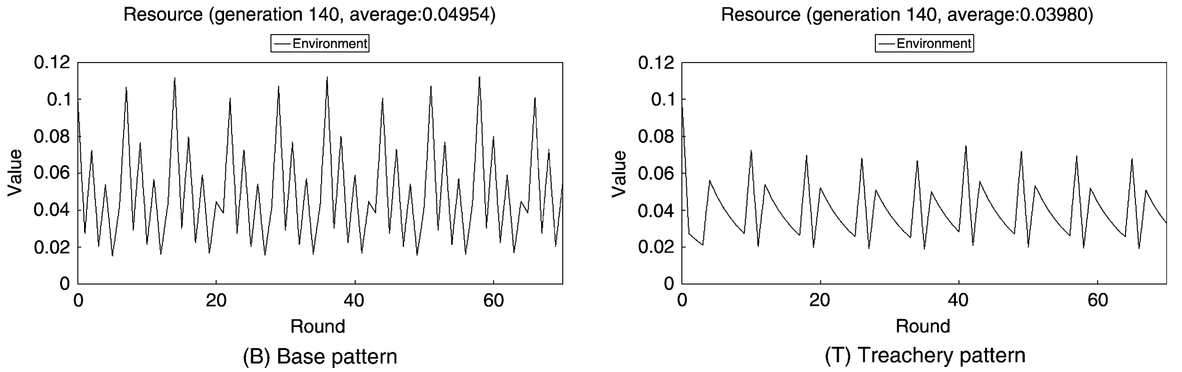

Fig. 20. Resource charts at early times (140th generation), corresponding to Fig. 19. Appearing in the titles are the average heights of the trees over all 400 rounds. (T) has a small average value, about 0.1.

Hence, the average scores of both players are low (about 0.1815 and 0.1987). On the other hand, Fig. 19(T) depicts a battle on another hill in the game-world of the same 180th generation. In this battle, one of the lumberjacks cuts the tree selfishly and gains a high profit, without waiting for the trees to grow. Here there exists a large difference between the average scores of the two lumberjacks (about 0.264 and 0.051).

The resource charts corresponding to Fig. 19 are given in Fig. 20. As we can see in these charts, the mean tree height of (T) is shorter because of the selfish actions of one of the two lumberjacks. However, judging from the corresponding state chart (Fig. 21), the state value of the selfish lumberjack (Fig. 21(T)) is higher than that of the other's. We thus see that if one lumberjack acts more selfishly, the game dynamics for each will be less productive, but the betrayer will increase his profit a little. This is a clear manifestation of the social *dilemma* game.

Now, let us study the evolution toward a competitive society. Fig. 22 displays the AGS diagram for the strategies of the two lumberjacks of Fig. 19(T), where one of the lumberjacks exploits the other. Here, the decision-making function of the exploited lumberjack is fixed, while the exploiting lumberjack's decision-making function is varied by changing the parameter θ_{21}. In Fig. 22(a), the attractor, represented by the state value, changes with the parameter θ_{21}. Here, the attractor consists of quasi-periodic motion

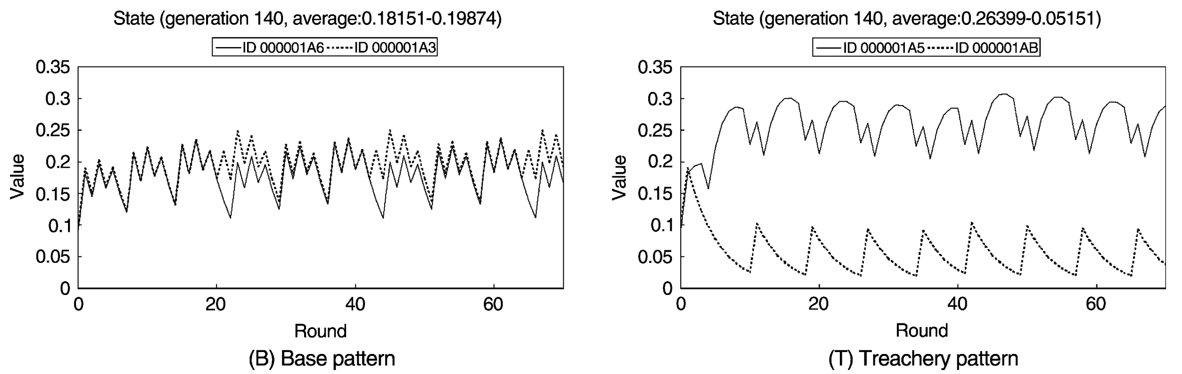

Fig. 21. State charts for the early times (140th generation) corresponding to Fig. 19. In contrast to the states of the two lumberjacks in (B), one of the two lumberjacks in (T) (the one who cuts the tree more frequently) maintains a higher state value, and the other remains at a very low state value.

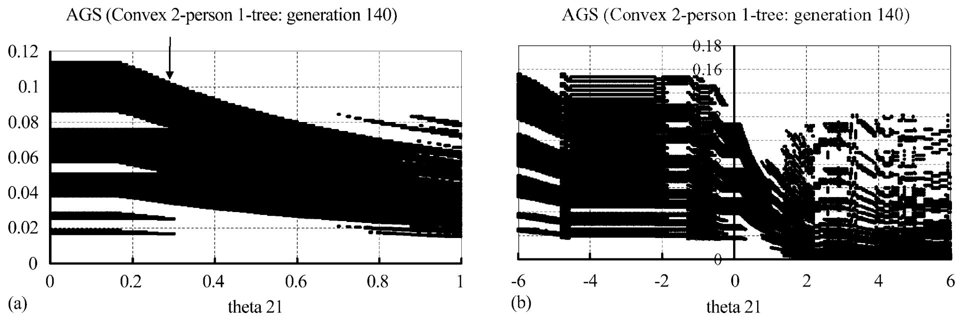

Fig. 22. AGS diagrams created by sampling the two players of the different two species in the same generation (the 140th generation). In each figure, the horizontal axis shows the value of the parameter θ_{21} in the decision-making function of a player, while the vertical axis shows the attractor of the tree height. (a) For all values of θ_{21}, the tree height changes quasiperiodically in time, taking infinitely many values. The range of these values changes with a continuous slope as a function of θ_{21}. The actual value of θ_{21} for the original player is 0.2879, as indicated by the arrow. (b) The same AGS diagrams over a wider range (from −6 to 6) of θ_{21} values. The attractor of the dynamics changes as the strategy of a player changes, and this change can be quite complicated.

(a torus) for all the values of θ_{21}. In contrast with Fig. 14, there is no flat region here, but the state changes with some slope when a strategy parameter is changed. The actual value of θ_{21} for the exploiting lumberjack is indicated by the arrow in the figure. In the region with a continuous slope in the diagram, such lumberjack that has larger θ_{21} by mutation cuts the tree more frequently, and has a higher score than others. Thus in the next generation, the mean value of θ_{21} increases, and the new standard for the game dynamics in the game-world goes down, following the slope structure in the AGS diagram. Since there is no flat structure here, this evolution continues to the bottom of the slope, where a selfish society is realized. (Such sloped structures are clearly seen in the AGS diagrams of selfish lumberjacks, as shown in Figs. 17 and 18.) The decline of cooperation through this mechanism clearly exemplifies the *tragedy* of the commons well—the process by which common resources gradually decrease due to the gradual increase of players' selfish behavior along with the evolution or "learning" of the players.

In Fig. 22(b), a blow-up of the AGS diagram discussed above is shown. it is interesting to compare this complex diagram with the clearly articulated AGS diagram (Fig. 14) when a cooperative society is established. We see that without articulated structure, it is extremely difficult for players to set up a certain type of game dynamics as a norm of cooperation. It follows that once the game-world falls into a competitive state, the society cannot easily get out of it over generations.

7.2. Linear LD games

To discuss the relevance of dynamics to the formation of cooperative society, we study the linear LD game, where the absence of articulate structure also has a dominant effect.

The *one-person* LD game was studied previously [1], whose relevant part to this study is briefly summarized in Appendix A. From the AGS of the one-person linear LD game, we can see that the game dynamics are in a quasi-periodic orbit over a wide parameter region, and this region consists only of a continuous, sloped structure (Fig. 26(b)). Plateaus exist only at the top of this sloped structure (where $x_d > 2/3$). This indicates that there are few plateaus in *linear* LD games, even in case of the one-person game.[12]

However, in the one-person game, cooperation with others is not necessary, and the existence of SSD, which is a prerequisite for the formation of cooperation standards in multiple-person games, is not relevant. Actually, in the one-person linear

[12] Note that, in the AGS diagrams of *convex* games, plateaus can be observed everywhere in any generation for the one-person game. By contrast, they do not exist in early generations of the two-person game, although they emerge in later generations of the two-person game through evolution.

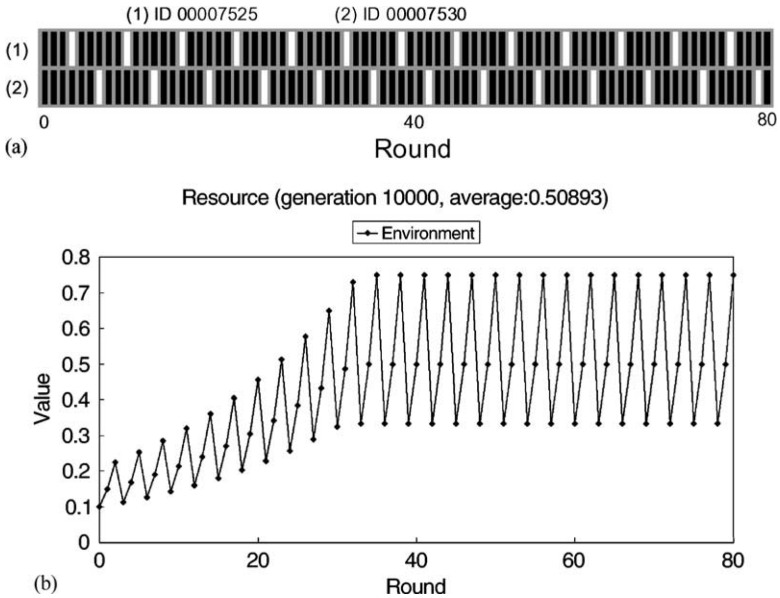

Fig. 23. Two-person, one-tree linear game (the 10,000th generation): (a) action chart; (b) resource chart.

LD simulation, the player starts climbing up the slope quickly, easily achieving optimal behavior [1], although there are only a few plateaus in the AGS.

In linear LD games with two or more persons, however, as stated in Section 4, there is a tendency toward competition in cutting a tree. Once the fitness value goes down, recovery is extremely difficult (Fig. 2(a1) and (a2)). This situation is due to slope-like structure that exists commonly in the AGS diagrams for the linear LD game. Here, the acceleration of the movement toward selfish behavior corresponds to the strategy parameter going down the slope in the AGS diagram. In the AGS diagrams of the linear LD game, there exists little structure that could provide a norm for cooperation. Therefore, the existence of the social dilemma characterizing the LD has a quite direct effect for the linear game. In this case, it is not easy for players from a non-cooperative society to realize SSD, which exists at the top of the slope. In the two-person linear LD game (Fig. 2(a1)), the players attain SSD after a very long time (approximately in the 7500th generation), and the cooperative society emerging then is stable. The game dynamics for this cooperative society are shown in Fig. 23. In contrast with this situation, in the case of the three-person linear LD game, the players are completely stuck in a non-cooperative society and are unable to crawl up to the SSD (Fig. 2(a2)).[13]

In the simulations we performed, random numbers chosen from a normal distribution were used to obtain the mutated parameter values in this study. Thus, the possibility for a great variation that could allow the players to climb up the slope exists, even though the probability of observing such behavior is small. What is to be noted here is that this probability becomes much smaller when the number of players increases from two to three. It should also be noted that, although in the fitness chart of the two-person game (Fig. 2(a1)), pulses in which the fitness value exceeds 0.01 can be seen occasionally, such pulses are very rare in the three-person game (Fig. 2(a2)). These results as a whole imply simply that ‘the effect of the number of players’ in systems plagued by the social dilemma is clearly and directly reflected here due to the fact that ‘no (or only little) articulation of the game-dynamics structure’ for the linear LD game.

[13] Simulations were conducted up to the 20,000th generation.

8. Dynamics-induced cooperation sustained in multiple person games

In this section, we discuss the importance of the dynamical nature of games in the maintenance of cooperation.

As stated in Section 4, the effect of the number of players does not work well in the case of the convex LD game. That is, in the three-person convex game, a cooperative society can be created at an early time (Fig. 2(b2)) and the mean value of the tree height is greater here than that in the two-person game. Thus in this case, the game environment can be maintained in a state of abundance. Once a cooperative society is established, it is maintained more stably than in the two-person game.

Fig. 24 displays the cooperative game dynamics that are dominant dynamics in the three-person convex LD game. The attractors of these dynamics consist of period-15 cycles from the viewpoint of each lumberjack (period-5 cycles from the viewpoint of the tree). After each lumberjack performs the five-action sequence of “cut, wait, cut, wait, cut”, they patiently wait for as many as 10 rounds, to let the other lumberjacks perform the same five-action sequence.

In contrast to the convex case, an increase in the number of the players for the linear LD game makes the realization of cooperation more difficult. From the perspective of the static game, both games are played by the same social dilemma. Then why does the difference in the tree growth rule result in such essential difference?

The answer to this question lies in the bifurcation structure of the convex game. For convex LD games, the productive, cooperative period-15 game dynamics in the three-person game has a greater stability, due to the mechanism resulting from articulation. In the three-person convex game, the period-15 attractor dynamics correspond to a plateau in the AGS diagram. This type of structure implies a greater stability against invasion of other strategies. The rate of cutting trees of this period-15 attractor, 2/15 for each player, is rather low, and the strategy here is therefore highly cooperative.

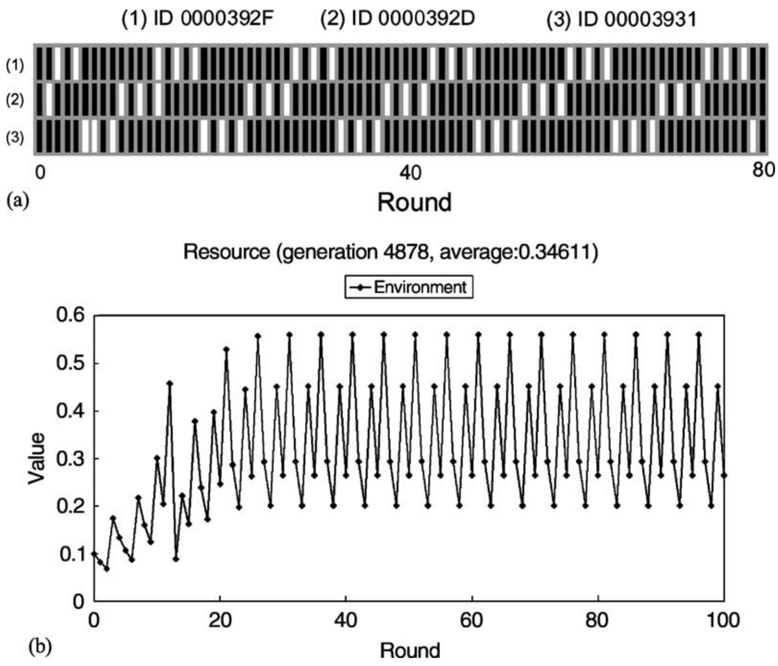

Fig. 24. The dominant pattern in a three-person, one-tree convex game (the 4878th generation): (a) action chart; (b) resource chart.

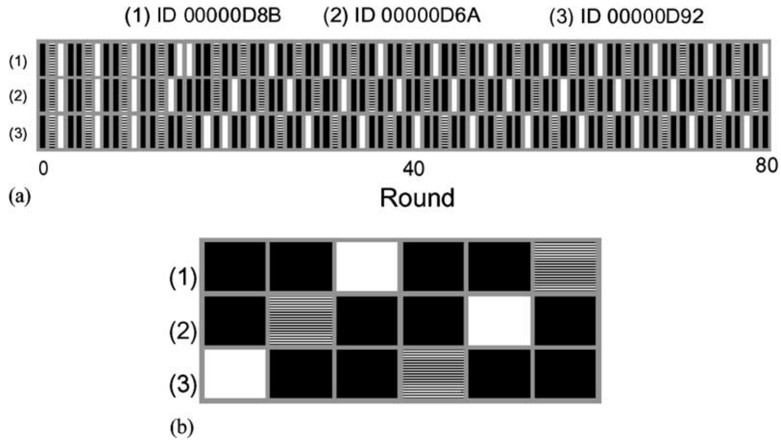

Fig. 25. The dominant pattern in a three-person, two-tree convex game: (a) action chart; (b) action sequence in the attractor part. Here, a black rectangle represents the action 'waiting', a white rectangle 'cutting tree 1' and a gray rectangle 'cutting tree 2'. Each tree is cut once every two rounds, and each player cuts one of the trees every three rounds. They control the growth dynamics of the trees by alternately their roles in both time and space (the trees).

However, in this three-person convex game it is difficult for a lumberjack to increase his gain if he deviates from the cooperation norms. Decision-making functions of lumberjacks in this game show that, from very early generations of the simulation, lumberjacks begin to adopt the principle in their strategies, "I cut the tree when my state value becomes too low". Therefore, if one lumberjack cuts the tree very frequently, the other two lumberjacks would acquire only little lumber each time they cut, and as a result, they too would also start cutting trees at a more escalated pace. At this point, because the period-15 dynamics is isolated from other stable dynamics, by articulation, the observed game dynamics would not change gradually, but, rather, would *jump* to an attractor at an extremely low level, if the frequency of tree-cutting increased too much. As a result, lumberjack cannot violate the cooperative norm completely in this three-person game, because such a violation can bring about his own loss. For this reason, the period-15 cooperation rule is highly stable and the high fitness value is maintained stably in this game. (A slightly distorted period-16 pattern can also be seen in some generations.) Here, the influence of the dynamical structure on the results, which cannot be considered in traditional, static game models, is sufficiently strong to overcome the tendency toward selfish behavior that results from an increase in the number of players (Fig. 25).[14]

Of course, it is interesting and important to study if the present mechanism for cooperation works effectively for more-than-three person convex games. By comparing the three-person case with the two-person case, there is no reason that the cooperation of the present mechanism will be more difficult for more person case, although there might be a threshold point at which the effect of the game dynamics meets that of the number of players. Although we have shown here that the former can be more important than the latter, future studies are necessary for more-than-three person games.

By contrast, in the case of linear LD games, little articulated structure in the game dynamics is formed, even in the case of a one-person game (see Fig. 26(b)),

[14] The relevance of dynamics to cooperation is also observed in the results on three-person convex LD games with different numbers of trees (one, two, and three trees). Among them, two-tree games exhibit the most profitable game environment, not three-tree games. This is also caused by the greater stability of a somewhat peculiar cooperation rule in the three-person, two-tree LD game, which yields period-6 game dynamics, as shown in Fig. 25.

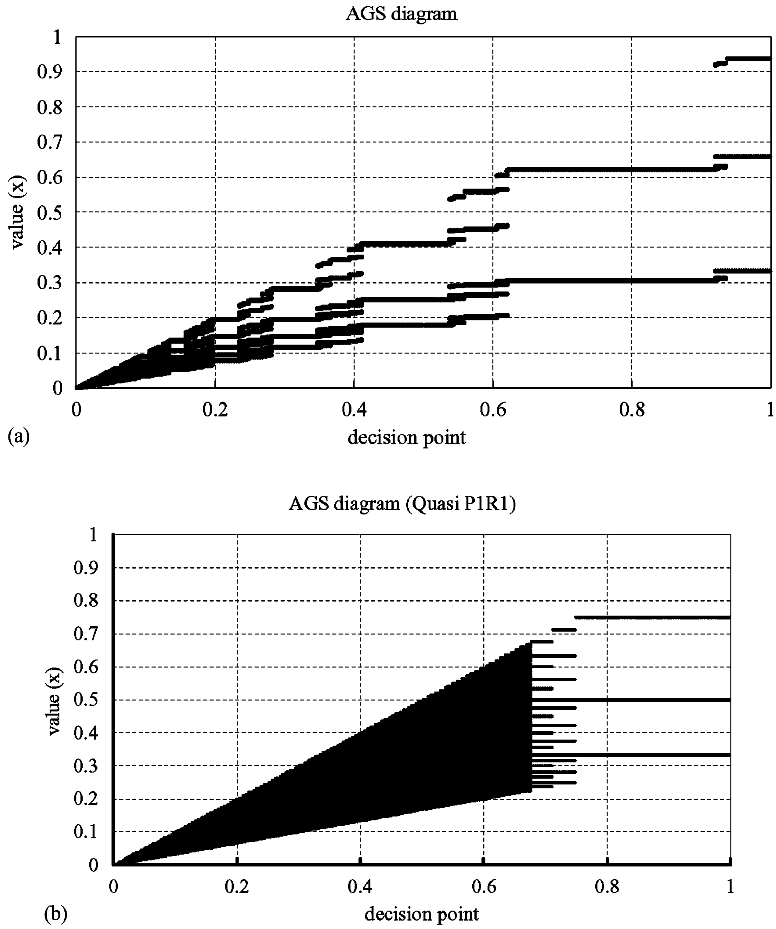

Fig. 26. AGS diagram of one-person, one-tree convex LD games: (a) convex LD game—change of the attractor with the strategy is plotted. A set of all the values of x taken by the attractor (i.e. all the values x takes between the 200th and 400th rounds), is plotted with the decision value, x_d, given in the horizontal axis. For example, the two parallel straight segments around $x_d = 0.8$ show that the dynamics of x is attracted to the period-2 cycle taking the values around 0.3 and 0.6 alternately. (b) The AGS diagram for the linear LD game plotted in the same way. Quasi-periodic attractors appear for $x_d \leq 2/3$.

because it is this structure that overcomes the tendency toward betrayal that the social dilemma necessarily involves, the effect of the number of players has the same effect in linear LD games as in the static game case.

9. Discussion

In this section, we discuss the findings for the LD game both in terms of the social dilemma and in terms of DS game.

9.1. Modeling of the social dilemma with static games and DS games

Examples of the social dilemma, including problems of garbage disposal, consuming pasture, and drains on various resources, can be modeled using static games, such as the n-person Prisoners' Dilemma, by extracting their common dilemmatic qualities. It is certainly important to direct attention to the common qualities among a variety of phenomena. However, should we model all the above problems in a similar manner?

For instance, there exists a significant difference at the level of dynamics between the games involving the consumption of resources that decrease only, like petroleum, and resources that are also produced under certain conditions. (The LD game is a model of the latter situation.) In DS games, these two situations are represented by different natural laws. These two types of game have completely different descriptions if they are modeled with DS games. When we model a game-like interaction as an algebraic payoff matrix of a static game, some kind of radical abstraction is definitely needed at logical level. In the course of such an abstraction, however, we may cut off what can only be described as specific dynamics, and it is possible that those omitted parts may be essential for the resolution of the social dilemma.

In the simulations presented in this paper, indeed, the above 'omitted parts' have an important effect on the avoidance of tragedy. In the modeling of the social dilemma using static games, such as the *n*-person Prisoners' Dilemma, it becomes increasingly difficult (or impossible) with the increase of the number of the players to avoid this tragedy. The effect of the number of players is undeniably real in any system characterized by the social dilemma. On the other hand, when we are faced with this dilemma in a real group or a real community, we do not always take tragedy-inducing actions. Why is this? As a new approach to understanding this problem, we introduced DS games. This is because we often adopt forms of cooperation accompanied with dynamics, as described below, when a tragedy is avoided in actual situations of social dilemma.

Our LD game exemplifies the social dilemma, and the realization of cooperation inherently becomes more difficult with the increase of the number of players. However, our study of the convex game shows that this increase does not always bring about a less cooperative society. Rather, the outcome depends on the nature of the dynamics exhibited in game. In other words, it is possible for the dynamical characteristics of the game to play a more important role than the effect of the number of players.

Indeed, as we have found, social norms that stabilize cooperation are formed from some dynamical structure of the system. Such norms in the convex LD game are formed as a result of articulation of an originally continuous distribution of strategies through bifurcation in the DS. With this articulation, each type of dynamics corresponding to each cooperative society is greatly separated from the non-cooperative dynamics. For this reason, a small change in the strategies of the players cannot destroy the cooperative society. It should also be noted that we often adopt some kind of norm for cooperation that is specific to the nature of the environment, when a tragedy is avoided under social dilemma in the real world.

On the other hand, it is difficult to form a cooperative society in the linear game, and it becomes more difficult with the increase of the number of players. In other words, in this case the increase in the number of players has the same effect as in the static game. Indeed, there is little or no articulated structure obtained through bifurcation. Without such articulation, the system is vulnerable to the invasion of selfish players, whose presence leads to an increase in the degree of selfish action throughout the game-world.[15]

Note that the emergence and maintenance of the cooperation in the convex LD games do not require any external force. Rather, this cooperation is created spontaneously within the system. The social norms for cooperation are developed through interactions among the lumberjacks and the game environment. Also, sanctions applied to those who violate the social standards are formed from the dynamical structure of the system. It is important to note this is not implemented as *sanction strategies* at the stage of modeling. These social norms autonomously retain their stability within the players' system in the form of a certain rules for cooperation. Conversely, when we attempt to set a norm from outside the system, it is desirable to set it so that it coincides with a stable rule formed naturally in the articulated structure of the DS game. In this case, this norm can be maintained steadily with the aid of the nature of the DS.

[15] Indeed, we face with this process of the collapse of cooperation in our daily life: "I cheat a little, the environment becomes a little worse, but I benefit a little, and if I cheat a little more . . . ".

9.2. DS game

We now point out two advantages in modeling with the DS game.

First, in the DS game the decision-making mechanism is represented as a 'simple DS'. In this study, the lumberjack's decision-making function is expressed as a linear combination of state variables. In spite of this simple representation, players can attain high scores by adopting non-trivial dynamics. For example, some strategies lead to stable cooperative behavior in which different types of dynamics are employed to repel different invading strategies. Furthermore, some strategies can successfully repel invasion by continuously infinite number of unrealized strategies. Also, during the evolutionary process, memory of past battles with selfish strategies retained to a certain extent. Note that the following advantages exist in the representation of a strategy by a simple DS: (1) it allows us to understand the features of a player's decision-making process as features of the equation in the decision-making function. (2) It allows the possibility of using DS theory, such as that regarding bifurcations, as demonstrated, for example, by the AGS diagram. (3) It allows suitable expression of strategies to deal with a continuously changing game environment.

Second, the DS game framework provides an analytical method and a new viewpoint that cannot be provided in the traditional game framework. Although the convex LD game and the linear LD game from the viewpoint of DS games correspond to the same *n*-person Prisoners' Dilemma from the view-point of static games. However, these two DS games that represent the social dilemma generate two qualitatively different types of social phenomena. The difference is brought about in the game dynamics of the games. We thus see that, several real social situations that have their specific dynamical features, which would be modeled as the same payoff matrix[16] when applied abstraction and described in the static game framework, are possible to exhibit completely different qualities by expressed respectively as DS games with concrete dynamics. Particularly, with regard to 'cooperation', which is a key element in the issues of multiple decision makers, cooperation that involves dynamics can be discussed only in the form of DS games. Through the result of the LD game simulations we conducted, we were able to observe how interacting players develop dynamical cooperation rules step by step. The mechanism of this development can be explained only from the distinct view of DS games. Furthermore, the stability of the cooperative state can be analyzed in terms of the stability of the game dynamics, while the mechanism governing the changes in the cooperative rules can be explained as transitions between metastable attractors in the AGS diagram.

9.3. The promise of forming dynamical cooperation to avoid tragedy in the real world and in the theoretical model

An example of avoiding tragedy in the real world is seen in the issue of pasturing in Section 1.1. In that case, in order to reduce the damage by cattle, strategies to move cattle from one place to another in accordance with the growth of grass were adopted.[17] In addition to this example, there are innumerable cases involving the social dilemma and the sharing of resources within a community [17], and most of their solutions such as allocation of a place or rotation of roles take into account their space–time structure.

Actually, when we attempt to avoid tragedy in consuming resources, we normally come up (consciously or unconsciously) to consider the growth dynamics of the pasture, the degree of the restoration of the land, and the nutritional state of cattle. If we wish to obtain a certain amount of resources without fail, we need to manage the dynamics of the resources, and for this reason it is necessary to create a certain norm for cooperation. Indeed, we often behave based on some norm for cooperation, which might be in the form of explicit rules or implicit rules. As a result, we may begin to take actions such as "raising the resources

[16] It may be the payoff matrix of, for example, the Prisoners' Dilemma, or the Chicken game, or the Battle of Sexes, or etc.

[17] However, it is now argued by specialists whether such strategies have actually effective or not [19].

and then consuming them together" or "raising the resources and consuming them alternately". These are nothing but the actions we found in our simulations. In the real world, consideration of space–time structure is important to avoid tragedy in the face of the social dilemma, because if we ignore this, we directly suffer the tragedy it can lead to. The cooperative states realized in our DS game are metastable solutions that reflect the nature of the concrete dynamics of the model.

In traditional game theory, one cannot study such cooperation in the form of dynamics. Not to mention, one cannot investigate the dynamical stability of the cooperative state. It can describe neither the temporal change of resources nor the effect of the dynamics of the game environment. Of course, one could model such a social dilemma using a static game by preparing strategies such as 'no grazing (cooperation)', 'grazing (betrayal)', and 'grazing of n-cows (n-degree betrayal)'. However, in order to handle the problems of social dilemma, it is important to consider the dynamics of the action taken based on the dynamics of the resources, for example, the timing used in allowing cows to graze, depending on the state of the pasture, the nutritional state of the cows, and the economic state of the herds.

In the DS games studied here, norms for cooperation are organized spontaneously, in the form of articulated structure of the strategy, which arises through the bifurcation of the attractor dynamics. The realization of this co-operation, as well as the nature of the norms it involves, strongly depends on the environmental dynamics. This should not be surprising, as, indeed, the importance of environmental dynamics in the history of human society has been increasingly stressed recently. Diamond [10] has discussed the generation of different types of civilizations in response to different ecological conditions, and in particular in response to the degree of resource diversity. At a much deeper and broader level, the school the annals in the history science seriously considers the interference between human actions and the change of the environment in space–time, as discussed in the epoch-making volume *La Méditerranée*, by Braudel [8].

In this paper, the results of our numerical experiment and their analysis in the LD game not only provide a new viewpoint regarding the problem of the social dilemma, but also demonstrate the possibility of using DS games as models that can investigate the essence of games observed only at the level of dynamics. Of course, the present DS game analysis is too simple to capture the complexity of the space–time evolution of our society. Still, as long as our world includes space–time structure by nature, DS games should provide a powerful theoretical framework to study the evolution of societies consisting of multiple decision makers.

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Appendix A. Attractors of the game dynamics and the 'AGS diagram'

In [1], we introduced the one-person LD game using numerical analysis and discussed in detail the phenomena seen in its evolutionary simulations. Let us discuss the basic nature of this game in simplified models of the one-person, one-tree LD games relating to this study. Here we investigate the effect of a change in the decision-making function on the attractor of the game dynamics.

We make two simplifications. First, in the situation we consider here, the player never refers to his state, y; that is, the player makes his or her decision by referring only to the size of the tree. Second, the player cuts the tree if the size of the tree exceeds a certain value, called the *decision value*, x_d. The decision value uniquely determines the time series of the phase (x, y). The attractor of the time series can be a fixed point, or consist of periodic, quasi-periodic, or chaotic

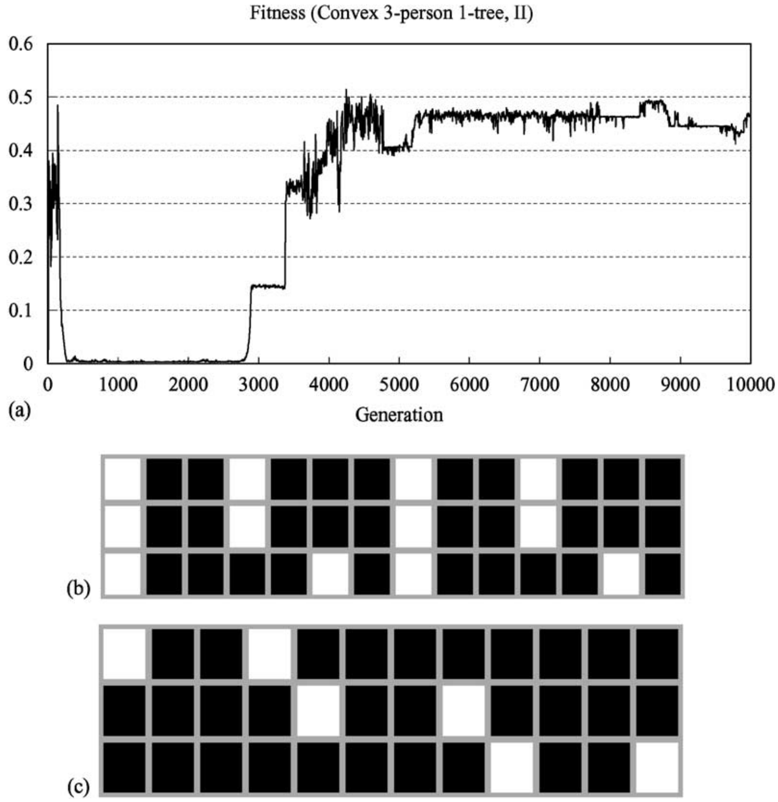

Fig. 27. Evolutionary process in a three-person convex LD game: (a) fitness chart for a three-person linear LD game; (b) action chart (the 3000th generation); (c) action chart (the 9000th generation).

motion. Its actual nature depends on the dynamical law (including the natural law) given to the system.

As in a 'bifurcation diagram', we have plotted the set of values of x in the attractor for values of x_d from 0 to 1 in Fig. 26(a). The figure is a diagram describing how the attractor of the game dynamics changes as a parameter in the decision-making function changes. We call such a figure the *AGS diagram*—describing transitions of the attractor of the game dynamics resulting from changes of the strategy. With the AGS diagram, one can study how the nature of the game dynamics shifts among various states (fixed point/periodic/chaotic game-dynamics, or productive/unproductive game-dynamics, etc.) with changes in the decision making. The following two characteristics of Fig. 26(a) are noted:

(1) For each decision value, the corresponding attractor is always a periodic cycle.
(2) There is an infinite number of 'plateaus' in which the attractors are unchanged over some range of decision values.[18]

For the one-person "linear" LD game, the AGS diagram shows that the dynamics are attracted to quasi-periodic motion if $x_d \leq 2/3$, otherwise to periodic motion (Fig. 26(b)).[19]

Appendix B. Another result of the three-person convex LD game

As mentioned in Section 4, in one of the three simulations of the three-person convex LD game, mutual

[18] For examples of such plateaus, see the period-2 and period-3 plateaus in Fig. 26(a).

[19] More detailed investigations of these two AGS diagrams in the evolutionary one-person LD games are given in [1].

tree-cutting society appeared at an early stage but the cooperative society was established and maintained later.

The corresponding fitness chart is given in Fig. 27 (a). This diagram shows that a cooperative society is created in a stepwise fashion from the competitive state, as in the two-person LD games. In Fig. 27(b), we have plotted the dominant pattern for cooperative behavior observed at the 3000th generation, in which the average tree height is approximately 0.05 and the average state of players is approximately 0.14. Fig. 27(c) shows the dominant action dynamics observed at the 9000th generation, in which the average tree height is approximately 0.45 and the average state of players is approximately 0.44. Both the game environment and the players' states are improved through development of cooperation rules.

As far as the observed results of three-person convex LD games are concerned, cooperative societies can stably be maintained, once they are formed at an early stage, as described in Section 4, or once formed through a development of cooperation rules eventually from betrayal societies, as in Fig. 27. Although we have not made detailed analysis using AGS diagram, the above results suggest that the cooperation is sustained through bifurcation of attractors as in the two-person convex LD game.

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