

Learning Shapes Bifurcations of Neural Dynamics upon External Stimuli

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Abstract. Memory is often considered to be embedded into one of the attractors in neural dynamical systems, which provides an appropriate output depending on the initial state specified by an input. However, memory is recalled only under the presence of external inputs. Without such inputs, neural states do not provide such memorized outputs. Hence, each of memories do not necessarily correspond to an attractor of the dynamical system without input and do correspond to an attractor of the dynamics system with input. With this background, we propose that memory recall occurs when the neural activity changes to an appropriate output activity upon the application of an input. We introduce a neural network model that enables learning of such memories. After the learning process is complete, the neural dynamics is shaped so that it changes to the desired target with each input. This change is analyzed as bifurcation in a dynamical system. Conditions on timescales for synaptic plasticity are obtained to achieve the maximal memory capacity.

Keywords: neural network, bifurcation, associative reward-penalty

1 introduction

One of the most important features of the brain is the ability to learn and generate an appropriate response to external stimuli. Output responses to the input stimuli are memorized as a result of synaptic plasticity. A wide variety of neural network models have been proposed to study how synaptic strength pattern is formed to memorize given input-output (I/O) relationships. In most of the studies[1], inputs are supplied as the initial conditions for neural activity, whose temporal evolution results in the generation of the desired outputs. However, recent experimental studies have shown that there exist structured spontaneous neural activity even in the absence of external stimuli[2]. Upon external stimuli, such neural activity is modified to provide appropriate response neural activities[3]. Considering these studies, we propose a novel perspective of the memorization of I/O relationships: If a neural system memorizes an I/O relationship, the spontaneous dynamics is modulated by a given input to provide a required output. In other words, the input changes the flow structure of the neural dynamical system to generate the corresponding output.

On the basis of this idea, we address the following questions: Can we construct an appropriate neural network model to demonstrate the learning process under biologically plausible assumptions? If so, under what conditions would learning be possible? What changes in the neural activity brings about the output when an input is applied?

2 Model

In modeling learning, we postulate the following two conditions that satisfy the biological requirements of the brain: (i) The learning process should not require detailed error information. In other words, the number of error information should be considerably smaller than the number of neurons. For example, the error back-propagation algorithm requires error information corresponding to each of the output neurons. In the case of biological learning with neural network, however, it is difficult to transmit such large amounts of information specifically to each neuron. (ii) I/O relationships should be learned one by one, i.e., the next novel I/O relationship is learned after one relationship has been learned with preserving the previously learned relationships. In contrast, in most neural network algorithms, many relationships are simultaneously learned by gradually changing the synaptic strength until all the relationships are memorized.

We introduce a layered network model consisting of input, hidden, and output layers along with a reinforcement learning such algorithm as the associative reward-penalty (ARP)(Fig.1A)[4], so that this model satisfy the above-mentioned conditions. In this model, several I/O relationships are learned one by one with only a single error signal which is given as the distance between the activity pattern of the output neurons and a prescribed target pattern. During the learning process, the synaptic strength varies in accordance with the Hebbian and anti-Hebbian rules, switched depending on the magnitude of the error signal.

To be specific, we adopt the following model with N neurons in each layer. Three types of synapses are considered: forward synapses (FSs), backward synapses (BSs), and mutually inhibiting intralayer synapses (ISs). FSs connect the neurons in the input layer to those in the hidden layer and the neurons in the hidden layer to those in the output layer. BSs connect the neurons in the output layer to those in the hidden layer, while ISs connect the neurons within a given layer (hidden layer or output layer).

The neural activity in the input layer is determined by an input pattern I , a vector whose element takes the value 0 or 1; the magnitude of the vector (input strength) is η (Eq.1). The neural activities in the other layers change, as shown by the rate coding model (Eq.2):

$$x_i = \eta I_i \quad (I \in \{0, 1\}) \quad (\text{input layer}) \quad (1)$$

$$\tau^{\text{NA}} \dot{x}_i = 1/(1 + \exp(-\beta u_i + \theta)) - x_i \quad (\text{the other layers}) \quad (2)$$

postsynaptic neurons as well as on the reward signal R determined by the error signal E , as

$$\tau^p \dot{J}_{ij}^p = R^p (x_i - r) x_j \quad (J \geq 0) \quad (p = \text{FS or BS}) \quad (3)$$

Here, r is the spontaneous firing rate (r is set at 0.1). The synaptic plasticity in this model has two characteristic features as follows.

(i) *Plasticity switched by the error:*

As mentioned earlier, the sign of R determines the synaptic plasticity. The sign of R changes with the magnitude of E such that

$$R^{\text{FS}} = \begin{cases} 1 & \text{for } E \leq \epsilon \\ -1 & \text{for } E > \epsilon \end{cases} \quad R^{\text{BS}} = \begin{cases} 0 & \text{for } E \leq \epsilon \\ -1 & \text{for } E > \epsilon \end{cases} \quad (4)$$

When the output pattern is close to (distance from) the target pattern, i.e., $E \leq (>)\epsilon$, the synaptic plasticity follows the Hebbian (anti-Hebbian) rule, as derived by substituting Eq.4 to Eq.3. The Hebbian (anti-Hebbian) rule stabilizes (destabilizes) the ongoing neural activity. In this manner, the error switches the synaptic plasticity between the Hebbian and anti-Hebbian rules. Note that under Hebbian rule, only the strength of the FS varies so that memories of the I/O relationships are embedded in the FSs.

(ii) *Multiple time scales:*

In most neural network studies, only two time scales are considered: One for neural activities and the other for the synaptic plasticity. Considering variety of timescales of the synaptic plasticity, we introduce two time scales for the plasticity of the FSs and BSs, τ^{FS} and τ^{BS} in Eq.3, respectively. As will be shown later, I/O relations are successfully memorized when the difference between the time scales is appropriate.

3 Results and Analysis

3.1 Learning Process

We first show that this model can be used to learn I/O relationships; the neural activity is varied when searching the target and stabilized when matching the target (Fig. 1B). When the error is large, the present neural activity becomes unstable by the anti-Hebbian rule and hence the neural dynamics itinerate among different patterns ($0 < t < 350$). The target pattern is searched during this iteration. We term this period the search phase in what follows. At $t \sim 350$, the magnitude of the error reduces to a sufficient extent when the output dynamics are within the neighborhood of ϵ . Once this occurs, the neural activity is stabilized as per the Hebbian rule, and the output activity remains close to the target ($350 < t < 800$). Thus, the activated synapses are continuously strengthened until a new target is presented. This period is called the stabilization phase. At $t \sim 800$, we switch a new input and the corresponding target pair to learn this.

At that time, the distance between the output pattern and the target pattern increases again, and therefore, the searching process progresses on the basis of the anti-Hebbian rule ($800 < t < 1200$). In this manner, the neural activity can reach the target by switching between anti-Hebbian and Hebbian rules alternately for synaptic plasticity, depending on the error. Now, we successively provide new input-target pairs after the time interval of stabilization phase, which is sufficiently long for learning an I/O relationship.

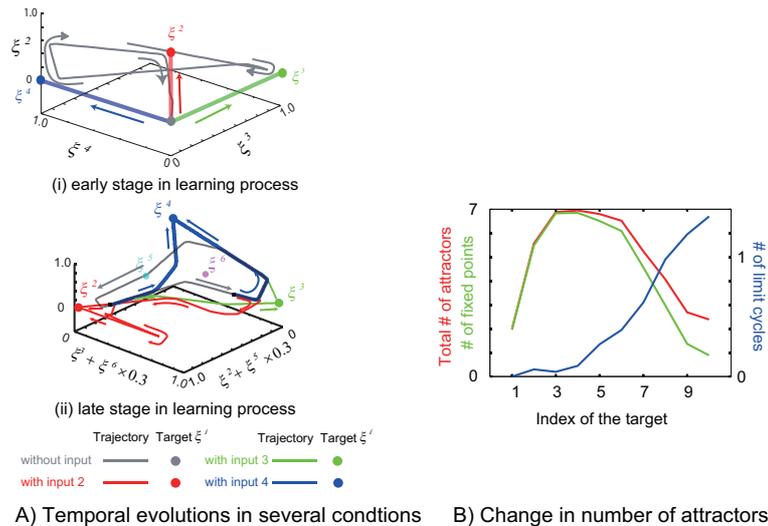


Fig. 2. Analysis of changes in the flow structure of the spontaneous neural activity during the learning process. A) Temporal evolution of neural activity in the output layer. Neural activity in the output layer is projected from the N -dimensional space consisting of neural activities in the output layer to three-dimensional space by obtaining the product of the output activity and the target pattern(s). (i) Neural dynamics after learning four I/O relationships. Each axis represents the product of neural activity and the corresponding target pattern. (ii) Neural dynamics after learning seven I/O relationships. Each axis represents the product of the neural activity and corresponding combined target patterns. B) Change in the number of attractors during the learning process. Numbers of fixed-point attractors (green line) and limit-cycle attractors (blue line) and the total number of attractors (red line) in the absence of inputs as a function of the number of learning steps, i.e., number of learned targets, are plotted.

3.2 Memories as Bifurcations

The learning process changes the flow structure in neural dynamical system both in the presence and the absence of external inputs. We examine the typical orbits of neural activity without any input and changes of such orbits with inputs in a dynamical system with a fixed synaptic strength after each learning step. Figure

2 shows examples of typical orbit in the attractor, as shown by the results of learning four and seven I/O relations. After four I/O relations are learned (Fig. 2A(i)), the neural activity in the output layer in the absence of any input is itinerant over three patterns that are close to three of the target patterns until the neural activity converges to a fixed point. Figure 2A(ii) shows an example of a limit-cycle attractor in the absence of inputs after seven I/O relations are learned, such that the orbit in the attractor (not a transient orbit as shown in Fig. 2A(i)) itinerates over the targets in the cyclic order 2, 6, 3, 4, 5. In both cases, these target patterns do not exist as fixed-point attractors without inputs. In application of the input, the fixed point and/or the limit cycle collapse and the corresponding target pattern change to a stable fixed-point attractor. Hence, this network memorizes these I/O relations as bifurcations. The number of memories varies through the learning process. Generally, as the learning progresses further, the number of memories increases in adequate time scales and that of fixed-point attractors begins to decrease, as these attractors are replaced by one or more limit-cycle attractors (Fig. 2B).

3.3 Memory Capacity: Dependence on Time Scales for Synaptic Plasticity

In this manner, our model can memorize I/O relationships successively up to some limit. The maximal number of memorized I/O relationships through learning process gives a memory capacity. This capacity depends on three time scales: τ^{NA} for changes in the neural activity, τ^{BS} and τ^{FS} for the plasticity of the BSs and FSs, respectively. From extensive numerical simulations, we found that the memory capacity reaches the maximal possible number N under the sparse coding case, when the condition $\tau^{\text{NA}} \ll \tau^{\text{BS}} \ll \tau^{\text{FS}}$ is satisfied. This implies that with a single synaptic time scale ($\tau^{\text{FS}} = \tau^{\text{BS}}$) as in the case of usual learning models, the capacity is not high.

To explain this time scale relationship, we study the synaptic dynamics during the search phase. In the phase, the output activity may come close to one of the previously learned target patterns. Since this pattern differs from the current target pattern, the attraction to it may be destroyed by the anti-Hebbian rule. In general, the longer the output pattern stays close to a state corresponding to a previously learned target pattern, the stronger is the destabilization of the state. Hence, the degree of destabilization of the previous memory increases with the residence time of the pattern on the corresponding state. As shown in Fig. 3, the residence time as a function of τ^{BS} decreases to a minimum at the τ^{BS} of the maximum memory capacity and increases as τ^{BS} approaches τ^{NA} or τ^{FS} . This can explain the dependence of the memory capacity on τ^{BS} .

Now we discuss the origin of this behavior of residence time. First, τ^{FS} determines the time scale of the memory decay, because the memory information is embedded in the FSs (Eq.4). Second, a smaller value of either τ^{FS} or τ^{BS} determines the time scale of the search phase, because the search for the target is possible only on the basis of the change in the flow structure by the anti-Hebbian

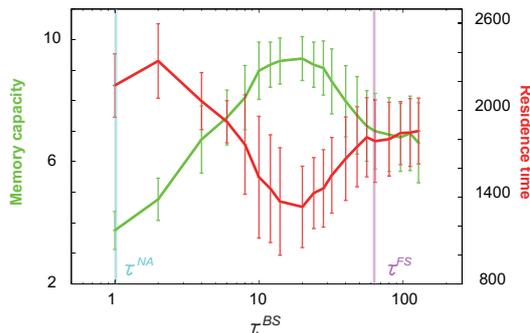


Fig. 3. Memory capacity (green) and residence time (red) as a function of the time scale of backward synapse τ^{BS} . The other two time scales τ^{NA} and τ^{FS} are fixed at $\tau^{\text{NA}} = 1$ and $\tau^{\text{FS}} = 64$. See text for the definition of capacity and residence time. Computed from the average over 100 learning processes for each τ^{BS} . Error bars indicate standard deviations.

rule. Since the search phase should be sufficiently smaller than the memory decay time, $\tau^{\text{BS}} \ll \tau^{\text{FS}}$ is required to preserve the previous memory during the search phase. Third, τ^{NA} determines the time scale for the neural dynamics under given fixed flow structure. If this value were larger than or of the same order of τ^{BS} , the phase structure would be modified before the neural activity change, so that the approach to the target pattern would be hindered. Hence $\tau^{\text{NA}} \ll \tau^{\text{BS}}$ is required. Accordingly the relationship $\tau^{\text{NA}} \ll \tau^{\text{BS}} \ll \tau^{\text{FS}}$ is required for the effective search for a new target without destroying the previous memory.

4 Discussion

In the present paper, we have proposed a novel dynamical systems model for the memory, in which the learning process shapes the “appropriate” flow structure of spontaneous neural dynamics, through successive presentations of inputs and the corresponding outputs. Memory recall is achieved as a result of the bifurcation of neural dynamics from spontaneous activity to an attractor that matches the target pattern induced by the external input. This bifurcation viewpoint is supported by, for example, a recent experimental study[3], in which the neural dynamics of the olfactory system of insects were studied in the presence and absence of odor stimuli. Here, we discuss how two above-mentioned features in our model can be implemented in our brain.

(i) *Plasticity switched by the error:*

In our brain, there exist several neural modulators such as dopamine, serotonin, norepinephrine, and acetylcholine. In particular, dopamine modulates the synaptic plasticity at the hetero-synaptic[5] and is projected to the cerebral cortex broadly. These natures correspond to the synaptic dynamics determined by

the product between the reward, the activities of pre- and postsynaptic neurons and that regulated by the error signal globally. In addition, the activity of dopamine neuron producing dopamine is related to evaluation how consistent the response is with the request[6]. Hence, in our brain, the switch between positive and negative plasticity corresponding to that between Hebbian and anti-Hebbian rules can be regulated by value evaluation through the concentration of dopamine.

(ii) *Multiple time scales:*

Because the synaptic plasticity is proportional to the product between the reward, the activities of pre- and postsynaptic neurons, the difference in the sensitivity of the reward signal or the activities of pre- and postsynaptic neurons controls that in the time scale of the synaptic plasticity effectively. In our brain, this sensitivity can be interpreted as the number and/or the type of receptors of neural modulators and neurotransmitters.

To achieve the maximum number of memorized patterns, a proper relationship has to be satisfied among the time scales of the changes in the neural activity and of the plasticity of the FSs and the BSs. By above discussion, the requested relationships for successful learning can be implemented by the distribution of neurons with adequate numbers and/or type of receptors in our brain.

According to our idea “memories as bifurcations”, the neural dynamics in the presence and absence of different inputs are distinct and separated because of bifurcation, which stabilizes distinct memorized patterns. In contrast to the view “memories as attractors”, our interpretation allows for the coherent discussion of diverse dynamics depending on the applied inputs. In fact, spontaneous neural activity is recently reported, which shows itinerancy over several states that are stabilized by inputs[7]. This gives remarkable agreement with our results. This bifurcation against the input strength will be experimentally confirmed by measuring the neural activity depending on the external stimuli.

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