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Natural language from function dynamics

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Abstract

A new approach to syntax and semantics of language is presented as a form of function dynamics, which is studied both analytically and numerically. The iteration of the function dynamics leads to articulation and formation of rules, which depend on each other. A hierarchy of meta-rules as rules of rules also emerges through the iteration when the initial function is suitably embedded. Iteration of a model with dialogue between two function dynamics is shown to generate a higher level structure.

Keywords: Natural language; Functional map; Meta-map

1. Introduction

The number of machines, which can output words like human beings, has been increasing these days. It was in 1950 that Turing proposed to consider 'Can machine think?' (Turing, 1950). Now, whether a machine can think or not, it can 'speak' language according to a program. If one could assume that the machine speaks with natural language only from this fact, the program in a brain (or somewhere else) could be regarded as substance of natural language. Indeed the formal language theory characterizes language by a rigid, programmed set of words and rules (Chomsky, 1959; Hopcroft and Ullman, 1979), and studies how sentences are generated by the set.

In the traditional context, natural language has been studied from the two aspects, syntax and semantics. The formal language theory focuses only on the former, and is not sufficient to be regarded as the theory of natural language. On the other hand, to study the semantics, the meaning of words or sentences has to be understood. There have been a lot of efforts to construct a theory for the semantics of natural language. Some concentrate on the relation between words and objects and others on the intentions of speakers.

The difficulty in the studying semantics mathematically is that it may be related to emotion, intention or something in our internal mental process. The semantics of natural language started from the study of the relation between words and objects, and then proceeded to the study of 'speech act' (Austin, 1960; Grice, 1989), which focuses

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on the relation between intentions and utterances and 'use' of language (Wittgenstein, 1953). Although these philosophical discussions often use mathematical logic to analyze the meaning of sentences, there have been few attempts to construct a mathematical model, which can generate sentences.

To construct semantics based on syntax theory, Montague (1974) has considered to be the modality of natural language seriously by adopting a higher-order modal logic to bring about the meaning into sentences, while Barwise and Perry (1983) have introduced mathematical description of situation and attitudes in which the speeches are uttered.

In this paper, we adopt a simple plan, which focuses on the circularity of words. If we try to construct a model of natural language without referring to a mental process, there are only signs (letters, words or sentences). In this situation, to capture a mathematical property of natural language, we have to focuses on a property of circularity; language is something described and something describing at the same time. Now, we assume that we can neglect the relation between words and objects, and pick up a network of words in which each word is defined by other words. (Note that a dictionary is an example of such a network.) We deal with this network, and study a possibility to construct a model of language as an 'autonomous' structure characterizing this network.

Of course, one might argue against this approach, pointing out that the correspondence with the real world is not taken into account. Although it is true that pursuing a structural relation between language and the real world is one relevant approach to language, it should also be noted that a clear border has to be put between the language and world in the beginning, once one adopts this approach. If the problem of language were formulated so, the study of language would totally depend on the structure of the real world.

For example, the formal language theory succeeded in classifying languages by disregarding the details of real natural language. In the approach of formal language theory, the relationship between words and the world which have been omitted has to be introduced later as a database, lexicon. Here, the relations between language and the real world appear as the number of nouns and verbs, the condition whether a verb requires an object or not, and so forth. Obviously we cannot determine the relations logically. These conditions are written in a dictionary. Accordingly, we need to add the dictionary to the syntactic structure, when we try to study natural language based on the formal language theory. In this sense, the semantics of natural language remains in the lexicon. As long as one separates the syntactic structure from the formal language from the knowledge of the real world in the beginning, the necessity of dictionary, untouchable from the theory, remains. In this sense, one abandons a serious attempt for a mathematical theory of natural language if one adopts the dichotomy of the syntax and real world.

As long as the model depends on a fixed set of words and rules, it cannot avoid viewing the language as combinatorics of a large set of words and rules within the society. It leads to the algorithmic view of natural language and it reduces the semantics to a lexicon. If words and rules are separated from the beginning, one can discuss neither their separation nor the classification of the sequence of words.

Accordingly we do not distinguish an object to be described from that to describe. We consider the transformation process of some objects that are put in and flow out. We try to avoid importing a detailed inner/mental process to model, since we are interested in the structure that the natural language should satisfy, not in the theory of brain. Hence, we consider an agent as simple filter transforming input into output disregarding the details of its internal process. Instead of introducing the complexity in the internal process in advance, we aim at reconstructing the 'internal process' from the behavior of the filter. The filter is an abstraction of a process of observing something and transforming it. The filter gives a configuration of a network giving a relation between inputs and outputs. The filter we considered is not fixed but can change in time, which leads to the change in the structure of language. Instead of introducing the real world as the database, we adopt an arbitrary filter initially, and investigate how the structure such as the 'articulation' and 'generation of rules' emerges from the circulation of processes through the filter that changes in time.

In this paper we use the word 'articulation' for the process of splitting the continuous world into discrete elements and interpreting them as the discrete words. As long as we do not presume the existence of a fixed set of (discrete) words, a theory is required that can deal with a generation process of articulation from continuous world. It will be shown that an agent with this 'dynamical filter' will perform articulation in this sense, and also that rules for this articulated objects, are also formed through the change of the dynamic filter.

To take into account of the circularity between the language and the world, words and rules are not separated in the beginning. We try to understand how the initial structure, given as 'inarticulate world', is transformed into a rigid structure with articulation and rules, through iterated use of language.

In this paper, we study the following features extracted as characteristics of the language, using this dynamic filter,

- articulation from continuous world;
- rule which operates over articulation;
- circulation between articulation and rule;
- hierarchy of rule and meta-rule;
- characteristic structure derived from dialogue, partly common and partly intrinsic to individuals.

To represent the dynamic filter mathematically, we adopt the dynamics of a function instead of variables, since what changes dynamically are not a state or an object, but the relationship to transform the input into output. We introduce the function $x \rightarrow f(x)$ as a filter from inputs (x') to outputs ($f(x')$). With this representation, the inseparability and circularity of words and rules are represented by a composition of the functions such as $f \circ f$ (a one filter) or $f^{\mathrm{self}} \circ f^{\mathrm{other}}$ (two filters) so that the relationship $x \rightarrow f(x)$ is applied to $f(x)$ itself. Here, the function $f(x)$ represents a network in which each element (x') is indicated ($f(x')$).

This paper is organized as follows. We propose models as a functional equation in Section 2, where a function for a function $f(x)$ not for a variable x, is introduced. Two models are presented corresponding to the situations that one agent observes itself and two agents observe each other. In Section 3, the dynamics of the one-filter (monological) model are investigated analytically. There, the function dynamics are represented by hierarchical combination of a one-dimensional map. The two-filter (dialogical) model is studied analytically in Section 4. The dynamics is represented by a coupled map among the hierarchical combinations in Section 3. In Section 5, we carry out computer simulations of these models to study the dynamics. Summary and discussion are given in Section 6.

Note we use here that the 'natural language' is not necessarily restricted to our language we speak or write. We use the term 'natural language' to represent a structure that cannot be solely described by a syntactic structure given by a formal system. With the function dynamics, we intend to introduce a general framework to study such structure mathematically.

2. Model

As an abstract model for language, we adopt a filter that transforms input into output. Each individual observes the outer world through this filter, which changes dynamically in time, through the interference between inputs and outputs. For simplicity, we adopt a one-dimensional map $f_n(x)$ (a function with a single variable) for this filter. In other words, input x is transformed into $f_n(x)$ through this filter, which is an abstraction of interpretation process. Within this framework, the initial function $f_0(x)$ has all information about the world, while we study how this filter $f_n(x)$ evolves in time, according to the iterations of $f_n(x)$, autonomously or dialogically. The rigid structure achieved through the iteration is studied and classified.

First, we start the case with one filter (one agent). Here we assume that the output from the filter is applied as an input to it, and the filter is changed so that its output is matched with the input (see Fig. 1a). For such evolution, we need to consider the dynamics of the function $f_n(x)$ depending on $f_n(x)$ and $f_n \circ f_n(x)$, in general. As a simple example to capture the dynamics of a filter with its self-reference, we study the following functional equation (Kataoka and Kaneko, 2000a);

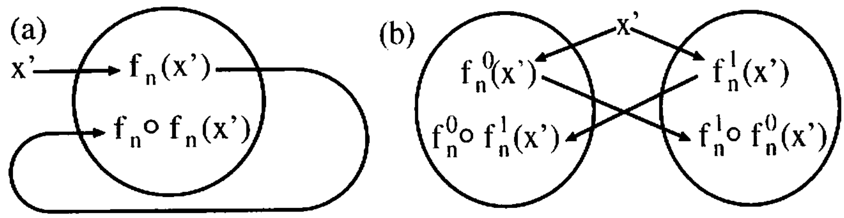

Fig. 1. (a) The schema of the model for one filter. $f_n(x)$ Interprets x' as $f_n(x')$ and observes it as $f_n \circ f_n(x')$. (b) The schema of the model for two filters. Each $f_n^i(x)$ interprets x' as $f_n^i(x')$ and observes the other's output as $f_n^{\rm self} \circ f_n^{\rm other}(x')$.

$$f_{n+1}(x) = (1-\epsilon)f_n(x) + \epsilon f_n \circ f_n(x) \qquad (1)$$

We choose the initial function $f_0(x)$ so that its range is smaller than the domain ([0, 1]), while ϵ is chosen to be $0 < \epsilon < 1$. As far as this condition is satisfied, $f_n(x)$ evolves in a bounded interval. Here, $f_n \circ f_n(x)$ is a self-referential term of the filter. Note that this self-referential term means that $f_n(x)$ is an operator and variable (operand) at the same time. It corresponds to be inseparability of word/rule and rule/meta-rule. In this model, the filter evolves so that it satisfies the relation $f_n(x') = f_n \circ f_n(x')$. This relation implies that the filter is fixed, if the self ($f_n(x)$) agrees with self-observation ($f_n \circ f_n(x)$).

For example, when one listens to a sound x' through the filter $f_n(x)$ and produces an output $f_n(x')$, the output is observed as $f_n \circ f_n(x')$ by the filter. In this example, the relation $f_n(x') = f_n \circ f_n(x')$ represents a self-consistent relation for an imitation of sound.

This function dynamics have fixed points x' satisfying $f_{n+1}(x') = f_n(x')$, which are classified into the following two types (Kataoka and Kaneko, 2000a).

- type-I fixed point, $x^I = f(x^I)$;
- type-II fixed point, $f(x^{II}) = x^I (x^{II} \neq x^I)$.

Here, we denote fixed point ($f_n(x') = f_{n-1}(x')$) as $f(x')$ without the suffix n. These fixed point conditions do not necessarily mean a fixed function as a whole, but are satisfied locally only at $x = x^I$ or $x = x^{II}$. Here, $f_n(x)$ intersects the identity function at each type-I fixed point, while at each type-II fixed point $f(x)$ has the same value as type-I fixed point.

This function dynamics can be split as.

$$\begin{cases} f_{n+1}(x) = g_n(f_n(x)) \\ g_n(x) = (1-\epsilon)x + \epsilon f_n(x) \end{cases} \qquad (2)$$

Thus, the evolution rule of a filter is decided from the filter itself. This map $g_n(x)$ acts as a rule for each point $f_n(x')$. We call this $g_n(x)$ a generated map. Following the notation for a fixed point, we denote the generated map determined by $f(x')$ as $g(x')$ without suffix n. In Section 3, we study this function dynamics for a few classes of initial functions.

Next, we study the case with two filters, by extending Eq. (1) to dialogical function dynamics. The two filters $f_n^0(x)$ and $f_n^1(x)$ face each other and accept and each other's output, and changes themselves according to the output (see Fig. 1b). As a simple extension of Eq. (1) we adopt the following form.

$$\begin{cases} f_{n+1}^0(x) = (1-\epsilon)f_n^0(x) + f_n^0 \circ f_n^1(x) \\ f_{n+1}^1(x) = (1-\epsilon)f_n^1(x) + f_n^1 \circ f_n^0(x) \end{cases} \qquad (3)$$

This is a minimum model with the observation of the observation of the other agent, which is represented by the second term $f_n^{\rm self} \circ f_n^{\rm other}$ in the right hand side of Eq. (3). This simultaneous functional equation cannot be split as given in Eq. (2). In Section 4, we study this function dynamics.

Since both the function dynamics are equations for the value of $f_n(x)$ (not for x), if some x' and x'' take a same value ($f_n(x') = f_n(x'')$) at n, the subsequent evolutions of $f_n(x')$ and $f_n(x'')$ are identical ($f_m(x') = f_m(x'')$ for $m > n$).

3. Dynamics of one filter

3.1. One-dimensional map

In this subsection, we study the case in which a

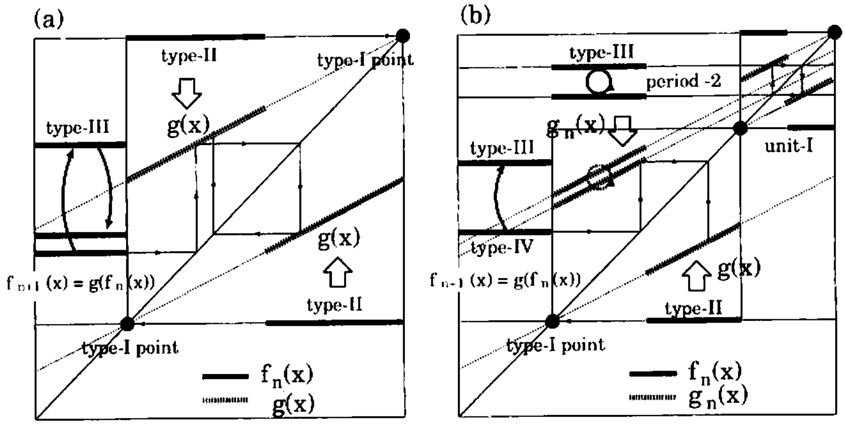

Fig. 2. (a) An example of a fixed generated map. Type-I fixed point specifies an area where a generated map can exist while type-II fixed points determine an actual generated map. Here type-I fixed points and type-II fixed points generate a fixed generated map $g(x)$. At the left part, there are type-III points, which is evolved by $g(x)$. (b) The first level meta-map given in Section 3.2. Unit-I specifies an area where a generated map can exist and the type-III fixed points determine an actual generated map. The evolution of type-III points on the top is determined by the fixed generated map in this unit-I. In this figure type-III points on the top evolve with period 2. These points generate a time-dependent (period 2) generated map ($g_n(x)$). The evolution of points on the left is determined by $g(x)$ and $g_n(x)$. In each figure, $\epsilon = 0.5$.

generated map $g_n(x)$ becomes a fixed function after some iteration. Note that a fixed generated map is derived from a fixed function $f(x)$, consisting of type-I and type-II fixed points. By rewriting $g(x)$ as $g(x) = (1-\epsilon)(x - f(x)) + f(x)$, $g(x)$ intersects a point $(f(x), f(x))$ and has slope $1-\epsilon$ The point $(f(x), f(x))$ is nothing but a type-I fixed point, and the intercept of $g(x)$ is decided by the type-I fixed point. The type-I point specifies an interval where $g(x)$ can exist and the form of $g(x)$ is decided by the arrangement of type-II fixed points. A type-II fixed point (x', $f(x')$) decides the actual form of the generated map $g(x') = (1-\epsilon)x' + \epsilon f(x')$ (see Fig. 2a).

We denote a domain of $g(x)$ as U^1. A point $f_n(x') \subset U^1$, with $x' \not\subset U^1$ evolves under a generated map. If a condition $g(U^1) \subset U^1$ is satisfied, $f_n(x')$ evolves under $g(x)$ within U^1. If $f_n(x')$ never falls onto a type-II fixed point but $f_n \circ f_n(x')$ is a type-II point, we call $f_n(x')$ type-III point. The type-III point evolves under the generated map $g(x)$, while the interval, which determines the dynamics of type-III points, is called 'unit-I'.

Using $g(x)$, we can embed a one-dimensional map in this function dynamics under some restrictions. This one-dimensional map $g(x)$ consists of several piece-wise linear maps with slope $(1-\epsilon)$. If type-I fixed points are located discretely, the generated map has slope $(1-\epsilon) < 1$ except for discontinuous points separating two piece-wise linear segments. Thus, for almost all points, the one-dimensional map has only periodic attractors with finite periods[1].

3.2. *Meta-map*

In the last subsection, a one-dimensional map $g(x)$ ($x \in U^1$) is embedded in function dynamics. Dynamics of the type-III points determined by $g(x)$ are within the interval U^1, if the type-II and

[1] If the function has a continuous interval in which $f(x) = x$, $g(x)$ is determined by the configuration of the part of $f(x)$ consisting of type-II points as $g(x) = (1-\epsilon)x + \epsilon f(x)$. Now the slope of $g(x)$ determined by $g'(x) = (1-\epsilon) + \epsilon f'(x)$ depends on x. Thus, we can embed a one-dimensional map with the slope larger than unity, and the generated map can produce chaotic orbits. The function $f_n(x)$ of type-III points can change chaotically in time. However, the initial condition with a continuous interval consisting of type-I points is exceptional initial condition in the functional space, and it cannot appear unless the initial function does not have such an interval. The details are in Kataoka and Kaneko (2000b).

the corresponding type-I points are located suitably. This process is constructed hierarchically. In this subsection, we choose a unit-I instead of a type-I fixed point and a type-III point instead of a type-II point; to see how $f_n(x)$ evolves under time-dependent $g_n(x)$ determined by the type-III point.

Since $f_n(x)$, consisting of type-III points, is time-dependent, the generated map, $g_n(x) = (1-\epsilon)x + \epsilon f_n(x)$ is also time (n)-dependent. The unit-I specifies an area where $g_n(x)$ can exist and an actual $g_n(x)$ is determined by type-III points (see Fig. 2b).

We define a unit-II in the same way as the unit-I. A domain consisting of type-I, II and III points is denoted as U^2. We call the domain U^2 unit-II if the condition $g(U^2) \subset U^2$ is satisfied. The point $f_n(x') \subset U^2$ for $x' \not\subset U^2$ evolves within U^2 under the generated map determined in U^2. Note that $g_n(x)$ is n-dependent and its evolution gives the rule how the one-dimensional map (that governs the change of $f_n(x')$) changes in time. In this sense, $g_n(x)$ gives a rule to change the map, and thus is called a 'meta-map'. The dynamics of the function $f_n(x')$ that evolves by unit-II, are determined by $g(x)$ or $g_n(x)$ therein. If $g(x)$ determines the motion of a point $f_n(x')$, $f_n(x')$ is a type-III point, since its dynamics are determined by a type-II point. We call $f_n(x')$ a type-IV point, if the evolution of $f_n(x')$ is determined by $g_n(x)$, since its dynamics are determined by a type-III point.

This construction of a meta-map in some intervals can be continued to a higher level, to produce a meta-meta-map and so forth. To proceed such a hierarchy, we define a type-N point and a unit-N as follows:

A unit-N is an interval U^N that consists of type-I, II, ..., $N+1$ points and satisfy a condition $g(U^N) \subset U^N$. A point $f_n(x') \subset U^N$ $x' \not\subset U^N$ evolves under the unit. The point $f_n(x')$ is a type-N point, if a motion of $f_n(x')$ is determined by $g_n(x)$, which is generated from type-$N-1$ points. Thus, $f_n(x')$ is a type-$N+1$ point if and only if $f_n \circ f_n(x')$ is a type-N point. For example, a type-II point produces a fixed generated map $g(x)$ and a point that evolves under the $g(x)$ is a type-III point. The type-III point produces $g_n(x)$ and a point that evolves according to $g_n(x)$ is a type-IV point and so forth. The meta-map $g_n(x)$ is constructed hierarchically, while a point evolves under the meta-map can change its type in time. The point generates a meta-map again and determines the dynamics of other points. The meta-map generated by type-I, II, ..., N is called a $(N-2)$th-level meta-map accordingly (note that the one-dimensional map generated from type-I, II was the 0th level meta-map).

A function consisting of type-N points is also denoted as $f_n(x) = f_n^N(x)$. Then, the function dynamics of the original equation is rewritten with respect to the type has the form.

$$\begin{cases} f_n^N(x) = g_n^{N-1}(f_n^N(x)) \\ g_n^N(x) = (1-\epsilon)x + \epsilon f_n^N(x) \end{cases} \tag{4}$$

Here, $g_n^N(x)$ is a generated meta-map from $f_n^N(x)$.

By suitably choosing an initial function, we can construct an arbitrary level meta-map. A point evolving by meta-map changes its type through iteration and syntax of rule is generated as a sequence of its types and corresponding fixed points. An example is shown in Section 5.

4. Dialogue with two filters

Here the dynamics of two filters following Eq. (3) is studied. Since Eq. (3) cannot be split into two parts $f_n(x)$ and $g_n(x)$, the analysis by the generated map in the last section is not valid. However, if these two filters have the same form $f_n^0(x') = f_n^1(x') = f_n(x)$ at an interval I and this interval satisfies the condition $f_n(I) \subset I$, the evolution of $f_n(x)$ is the same as Eq. (2) in this interval. Here we assume the existence of such interval I that $f_n^0(x) = f_n^1(x)$ holds for $x \in I$ (with $f_n(I) \in I$). The existence of such interval in the initial function may not be an absurd assumption, since dialogue is thought to be impossible unless agents have some common structure for their cognitive process. In fact, if we choose completely different initial functions for the two agents, the evolution of Eq. (3), in most cases, leads to a trival fixed point function, $f(x) = $ constant, for all x.

The function within the interval evolves as is studied in the last section. On the other hand, for a point x' satisfying the condition $f_n^0(x'), f_n^1(x') \in I$, $x' \notin I$. Eq. (3) is rewritten as:

$$\begin{cases} f_{n+1}^0(x') = (1-\epsilon) f_n^0(x') + \epsilon f_n \circ f_n^1(x') \\ f_{n+1}^1(x') = (1-\epsilon) f_n^1(x') + \epsilon f_n \circ f_n^0(x') \end{cases} \quad (5)$$

This is a coupled map (Kaneko, 1993) with nonlinear time-dependent map $f_n(x)$. Two filters are coupled through $f_n(x)$. Here, a hierarchical class of $f_n(x)$ is given in the last section, while the evolution of $f_n^i(x)$ is bounded within the interval I. An example of numerical simulation is shown in the next section.

5. Numerical simulation

Previously we carried out numerical simulations of Eq. (1), starting from several continuous functions or random functions (Kataoka and Kaneko, 2000a,b), to demonstrate how type-I and type-II fixed points are generated, how higher-type points are formed. In this section, we study the function dynamics Eq. (3) of two filters, with the aid of computer simulation. Here, the domain and range of $f_n^i(x)$ are chosen to be [0, 1]. For the computer simulation, the interval is divided into M mesh points $(i/(M-1),\ i = 0, 1, \ldots M-1)$ with $M = 6000$. The initial function $f_0(x)$ represents information on the external world. Since no specific information is given here, we choose a random initial condition homogeneously distributed over [0, 1/2]. Following the discussion in the last section, it is assumed that each filter has the same initial form for a part of the interval. In the simulation, $f_0^0(x) = f_0^1(x) = f_n(x)$ is assumed for $x \in [0, 1/2]$, while $f_0^0(x)$ and $f_0^1(x)$ take different random patterns for $x \in [1/2, 1]$. Thus, for $A \equiv [0, 1/2]$, $f_n(x)$ evolves as studied in Section 3, while for $B \equiv [1/2, 1]$, $f_n^i(x)$ evolves as a coupled map with the coupling map with the coupling through the $f(x)$ $x \in A$, as studied in Section 4.

In Fig. 3 the snapshot $f_{50\,000}^0(x)$ is plotted, while the close-up of two-filter part ($x \in B$) is given in Fig. 3b, where $f_n^0(x)$ for $50\,000 \le n \le 50\,050$ are overlaid. Here, the length of transients is on the order of 10^2 at the interval A, and on the order of 10^4 at the interval B (for the present mesh size), and the plotted dynamics is already at an attractor.

In each interval, the functions are time-dependent for some points. This means that the generated map has periodic attractor and determines an evolution of some points. To see the characteristic motion, return maps of each interval are plotted in Fig. 3c and d. The points in the left inset of Fig. 3c consist of lines with slope $1-\epsilon$. This generates a one-dimensional map and acts as an evolution rule as discussed in Section 3.1. The points in the right inset, on the other hand, form a meta-map. In fact, for the points indicated by arrows, the return map is no longer single-valued and has two values. This is because the generated map therein consists of type-III points and leads to time-dependent $g_n(x)$. For the two-filter interval B, the return map given in Fig. 3d, covers the region of values which the return maps in A take. This implies that coupled map dynamics uses a meta-map in A.

The time evolution of type for a given point $(x \in A)$ is plotted in Fig. 4a. This point evolves under the meta-map and evolves changing its type. The change of type is periodic with the period 34 in the figure. In each point in A, there is characteristic sequence of types, that forms a syntactic structure for the dynamics of $f_n(x)$.

The motion in the interval B is characterized by defining a type in the same way as types in A. If $f_n^i(x') = y(x' \in B\ y \in A)$ and $f_n^i(y)$ is type-N point, $f_n^i(x')$ is a type-$N+1$ point. Since $f_n^i(x) \subset A$, the maximal type in B is $K+1$, if the maximal type in A is K. The minimal type in B is II, while type-II point is not necessarily a fixed point, since $f_n^0(x) = f_n^0 \circ f_n^0(x)$ is not a sufficient condition for a fixed point, as long as $f_n^0(x) \neq f_n^1(x)$. In Fig. 4b, the evolution of type for a given point $x \in B$ is plotted during the transient the time steps. The evolution of types shows that $f_n(x')$ $(x' \in B)$ uses a higher type (≥ 2) motion in A.

6. Summary and discussion

In the present paper, we have studied an abstract function dynamics to discuss generating process of words and rules, by focusing on how syntax is formed from the circularity. Within the function dynamics, a one-dimensional map can be embedded, from which a meta-map to change the map itself is generated hierarchically. Interaction of such function dynamics shows that the two agents, referring to their common part, generate a sequence of functional values with a specific syntactic structure. Here, we discuss the relevance of

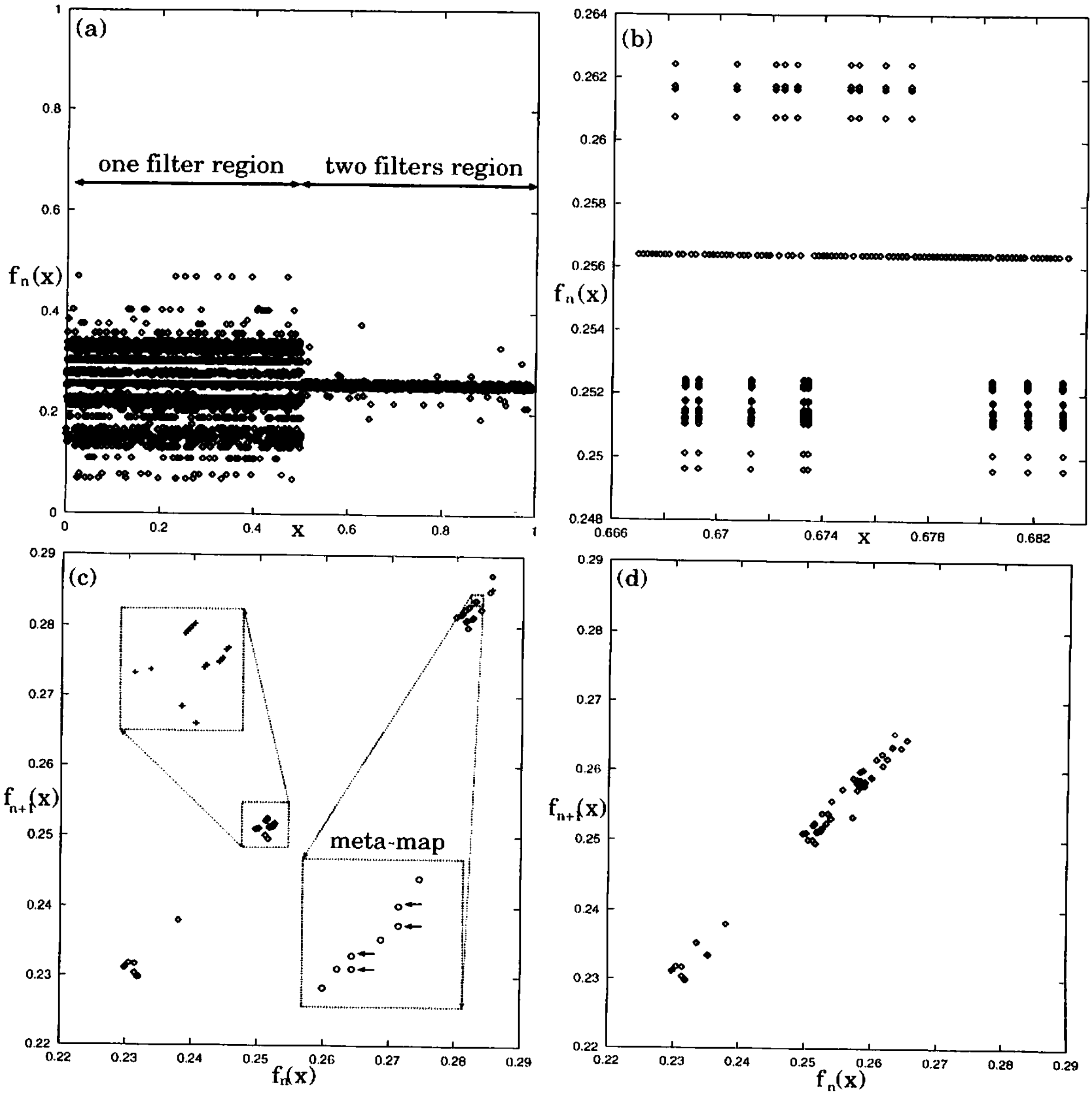

Fig. 3. (a) $f^0_{50\,000}(x)$, $c=0.02$. The initial function is chosen as described in the text. For $A=[0, 1/2]$ $f^0_n(x)=f^1_n(x)$, while for $B=[1/2, 1]$ the function evolves according to a coupled map Eq. (5), generated in $A=[0.0, 1/2]$. (b) $f^0_n(x)$ for $x\in[40/60, 41/60]$ are overlaid for $50\,000\leq n\leq 50\,050$. If there is a single points for given x, the corresponding $f_n(x)$ falls onto a fixed points, while $f_n(x)$ is a cycle with period P, if there are P points for x. (c) Return map $(f_n(x), f_{n+1}(x))$ is plotted for $50\,000\leq n\leq 50\,050$ for the interval A (the inset is a close-up of the corresponding region). (d) Return map for the interval B.

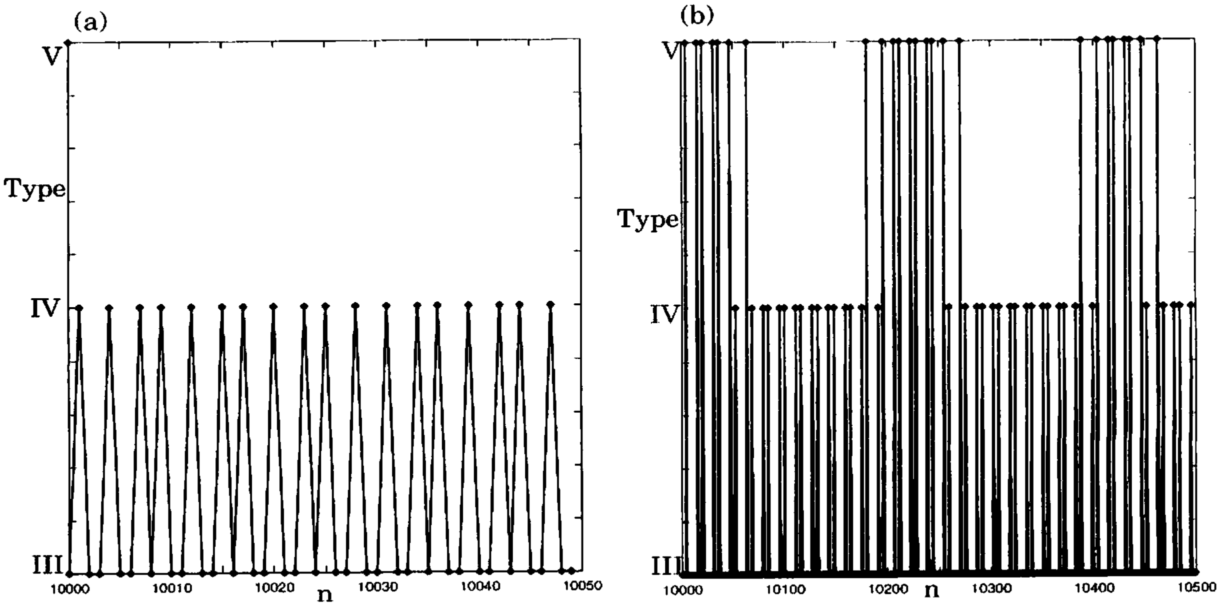

Fig. 4. The time series of the type for a given $f_n(x')$ ($x' \in A$) for $10\,000 \le n < 10\,050$, which evolves under the meta-map (given in Fig. 2c). Here $f_n(x)$ has a period 34. (b) The time series of the type of $f_n^B(x')$ for a given $x' \in B$ ($10\,000 \le n \le 10\,500$). Although $f_n(x)$ at $x \in A$ is already fallen onto a periodic attractor, the coupled map dynamics in B is still in transient (i.e. dialogue process in proceeding).

our results to the target problems presented in Section 1.

6.1. *Articulation*

For a given value $a = f_n^A(x)$ the inverse set $I_n \equiv \{y \mid a = f(y)\}$ is given as an articulated class. This means that the filter articulates the continuous world x into some segments according to the value $f_n^A(x')$. With the evolution, $I_{n+1} \supset I_n$, until the dynamics of this system is given as a set of relation among these intervals as $n \to \infty$. This reduction of degrees of freedom from a continuous world to discrete elements proceeds to so that the elements satisfy the self-consistent relation $f_n(x) = f_n \circ f_n(x)$. This articulation is most clearly seen in the relation between type-I and type-II fixed points. Intervals of type-II points corresponding to the same type-I fixed points are generated from an initial continuous function.

6.2. *Rule, meta-rule, and inseparability of articulation and rules*

In Section 2, as a basic structure of the function dynamics, two types of fixed points are shown, generally obtained through the evolution. In the formulation in Section 3, a one-dimensional map is embedded based on the fixed points, leading to type-II points depending on a map generated by several type-II points.

The fixed point is an invariant part for iterations and can be regarded as the elementary 'word' which is a concrete base of the description. It is expected to correspond to nouns, names of objects. A one-dimensional map determines an orbit of type-III points, based on a set of type-II points. Thus, an orbit determined by the one-dimensional map can be regarded as a representation of the set of the 'words', and the set of 'words' can be regarded as 'higher-order word' representing the operation. We call these higher-order words as 'notions' here. Examples of such 'notions' are a verb, a noun representing abstract idea. They are words, which represent relation among elementary words.

Similarly, the first level meta-map determines an orbit of type-III and IV points, as an operation over a set of type-II and type-III points. Int this hierarchical configuration, each orbit is characterized by a sequence of types and a sequence of value $f_n(x)$. A point evolving under the Nth level meta-map evolves among type-III, IV, ..., $N+3$. The sequence of types represents the operation over elements, which belong to various types in the hierarchy. The change of types is expected to characterize a modality of the word, because it represents a sequence of words, which belongs to various levels (types) of the hierarchy of words.

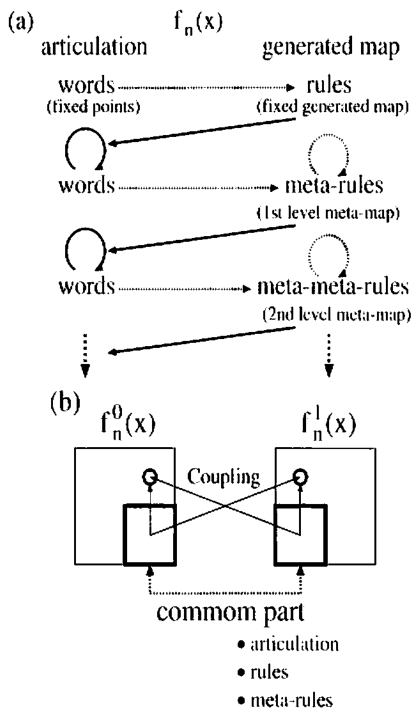

Fig. 5. (a) A schematic representation of the hierarchy of words and rules in this system. Lower-level words provide rules, which determine 'notions'. (b) A schematic representation of dialogue model. In the common part, words and rules emerges from the circularity as shown in (a). With the coupling through the common part, each $f_n(x)$ has a more complex syntactic structure, which cannot appear in a one-filter model.

Then the sequence of the values is expected to represent the word, which belongs to the modality determined by the type.

A map and a meta-map determine an orbit of a higher level. Hence, a higher-level structure is formed based on the lower-level structure, which we believe is an important characteristic in the language. For example, syntax over words is generated in our system, depending also on the words themselves. In Fig. 5a, the hierarchy of words and rules is shown.

6.3. Structure from dialogue

In the function dynamics for two filters, it has a higher level structure based on a part where both filters are the same (Sections 4 and 5). The term $f^{\text{self}} \circ f^{\text{other}}$ represents the interpretation of the world through the interpretation of the other filter. This interaction term leads to complex dynamics for the 'two-filter part' satisfying $f^0_n(x') \neq f^1_n(x')$ $(x' \in I)$, based on the meta-dynamics at a common part with $f^0_n(x') = f^1_n(x')$ $(x' \in I)$, as given in a higher level dynamics shown in Fig. 3b. Through the dialogue, a new structure is generated that cannot be generated from one filter. The function corresponding to this new structure evolves by a part that has common $f_n(x)$ and operates over generated maps and meta-maps. Here, the common part has a hierarchical structure with various types of points. If we interpret the orbits at the common part as 'notion' for agents, the coupled map dynamics give an operation over the sets of 'notions'. The coupled map dynamics as functional form determine syntax over the 'notion' and generate a sentence (see Fig. 5b).

Extension of the two-filter model to many-filter models is straightforward. Depending on the topology for interaction, one can, for example, embed a coupled map lattice or a globally coupled map (Kaneko, 1990, 1993). Here, an initial function consists of common part and 'N-filter part' (N is number of agents). The dynamics of $f^i_n(x)$st the common part can be regarded as the common 'notion' for all agents, while $f^i_n(x)$ at 'N-filter part' can be regarded as the operation over the sets of 'notions' among N agents. With this study, it is hoped that a mathematical framework is constructed to deal with social, rather than private, language.

In this paper, we have studied a network as whole and investigated the possible structure it organizes, and shown the fabrics of rules observed as the behavior of the filter. In our dynamics, all functional values are updated parallelly. Each utterance of agents is not proceeds successively. To consider such process, we need to change the function sequentially together with the temporal sequence of $f_n(x')$ values. Such extension is considered to be with the use of sequential dynamics, in addition to the noise to smooth ruggedness in $f_n(x)$. Some preliminary studies suggest that the basic structures presented in this paper are presented with the extension. Details of the syntax of 'sentences' used as language in this system should be studied in the future.

Non-trivial sets of functions on functions are studied in domain theory (Plotkin, 1983; Soto-Andrade and Varela, 1984; Rosen, 1991), where consistent sets including $f \circ f$ for all functions are constructed. The most significant difference between systems studied in domain theory and our system is the dynamical aspects of functions treated only in our approach. On the other hand, our meta-map is restricted within some intervals and is not the function on the whole interval. However, such a contraction can be removed in more general function dynamics. It will be important to bridge between dynamical system theory and domain theory, to study the semantics of natural language. If such theory is established, we can treat operations over operations in a single system. Our preliminary study of the function dynamics may provide the first step toward the study of dynamical domain theory.

In this paper, we have presented a minimal model for the interpretation of the world by a dynamic filter, and shown that its dynamics can be described as hierarchical structure. Generation of hierarchical description through iterations would be important, not only for the natural language, but also for all autonomous systems.

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