Constructive complexity and artificial reality: an introduction

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Abstract

Basic problems of complex systems are outlined with an emphasis on irreducibility and dynamic many-to-many correspondences. We discuss the importance of a constructive approach to artificial reality and the significance of an internal observer.

1. Introduction

For over hundred years, scientists, above all, physicists have tried to understand the complexity of nature, by decomposing it into simple processes that can be dealt with by simple theories. This strategy is often referred to as "Ockham's razor". It was regarded as ideal, in modern science, to describe a system in terms of a small number of parameters, variables, equations, etc. In a strict sense, however, the reduction to a system with a small number of degrees of freedom is not always possible. Then, one may introduce a "noise" term instead, in order to take residual degrees of freedom into account.

In statistical physics, this reduction was successful because of the introduction of appropriate order parameters. Even when a system has many degrees of freedom, it can often well be described by a macroscopic order parameter with a corresponding noise term, as can, for example, be seen in equilibrium statistical mechanics,

linear-response theory, system-size (omega) expansion [2,3], and slaving principle [4].

A related paradigm is the use of a "mode". Here a system is assumed to be represented by the superposition of some modes, like the Fourier modes. In solid-state physics these are attributed to some excitation, termed as "—on's". The use of modes is powerful as long as the system can be approximated by a linear one. In dynamical systems, it is also successful, even if the system is nonlinear, as long as it is not chaotic. It can be employed, for example, in quasiperiodic motion on a torus and in the representation by solitons.

The *reductionists'* pictures have been challenged by the discovery of chaos. First, the amplification of a tiny perturbation in chaos implies that the separation between microscopic and macroscopic levels is no longer possible. Second, the picture of "modes" is not straightforwardly applicable to chaos: Even if a system has just three degrees of freedom, it can implicitly

include continuously many modes. For example, a chaotic system cannot be represented by a finite number of Fourier modes. A one-to-one correspondence of simply chosen coordinates (such as Fourier modes) to original variables is no longer valid here. Complexity in grammatical rules to characterize chaos is discussed in detail by **Crutchfield** in the present volume.

Another challenge to the traditional picture can be found in a system called spin glass, which originated in the statistical mechanics of spin system with random interaction [6]. In relation with the phase transition problem, physicists searched for order parameter(s). However, a detailed theoretical analysis shows that the order in the low-temperature phase is represented only by a functional of order parameters, rather than a finite number of them. Indeed, in the neural network model based on the statistical mechanics of nonhomogeneous spin systems (often called as Hopfield model), a one-to-one correspondence between an interaction code and an attractor is no longer valid, when the number of stored input patterns is larger than a certain threshold. In spin glasses, the correspondence between an interaction and an attractor (or a thermodynamically metastable state) is highly complex, while, the sample dependence (i.e., dependence on the choice of couplings) remains finite even in the thermodynamic limit.

Thus the one-to-one correspondence between states and representations is challenged statically by spin glasses and dynamically by chaos. We need some framework to deal with the dynamical change of relationships among elements.

2. Logic for dynamic many-to-many correspondences

Let a system be composed of many dynamic elements, and chaotic motion be assumed in the system. Then any perturbation put in one element can be transmitted to other elements with amplification. In this situation, a chain of causal

relationships can bring about unexpected results [12,9]. (A Japanese proverb for such a 'strong' causal connection is "If the wind blows strongly then (finally) bath tubs sell well" [12,9].) The reasoning is the following: (1) If the wind blows strongly, the number of blind people increases due to the dust entering in their eyes. (2) If so, they try to earn money by playing a 'shamisen' (a traditional Japanese musical instrument made of cat's skin). (3) If so, the demand for 'shamisen' increases. (4) If so, cats are hunted recklessly. (5) If so, cats extremely decrease in population. (6) If so, mice increase in population. (7) If so, (wooden) tubs are gnawed by mice. (8) If so, tubs are sold well.

One of the authors (KK) encountered such situations when working with coupled map lattices [16] or networks of chaotic elements [17]. In a coupled map lattice with chaotic dynamics, a tiny perturbation at a lattice point is amplified to nearby elements. A macroscopic order corresponding to the dissipative structure [5] can appear, but again be destroyed by the chaotic dynamics until a next ordered structure appears. In a network of chaotic elements, clusters of synchronized oscillations may appear. Identical elements can differentiate due to the orbital instability in chaotic dynamics. This mechanism, called (dynamical) clustering, is commonly seen in globally coupled dynamical systems. Futhermore, the members of a strongly correlated group change in time, leading to a ceaseless change of relationships (see **Kaneko** in the present volume). Clustering and collective behaviors can also be seen in globally coupled oscillators as studied by **Nakagawa**. The synchronization between external (limb's) and internal (neural) oscillations is essential to the model of bipedal locomotion by **Taga** in this volume.

Besides in chaos research [7], the use of one-to-one correspondence has been challenged in many branches of science. In brain science, the hypothesis of a grandmother cell has been doubted. Such doubts have led to the notion of a distributed representation of information,

and the research of neural networks related with the spin glass theory. However, such challenges remain at a static level. In contrast with the static logic, the necessity of a dynamic logic has been postulated by Malsburg, Vaadia, Aertsen, Dinse, Freeman, one of the authors (IT), and so on (see the papers by **Tsuda**, **Aertsen**, **Dinse**, and **Freeman** in the present volume).

In an ecological system the necessity of a logic *that grasps a complex system without reduction to an ensemble of simple elements* has been stressed by Elton [8]. Kawanabe has pointed out the necessity of a logic to represent the above Japanese proverb. Indeed it is known that in some ecological systems there are keystone species, a removal of which strongly damages the whole ecosystem [1].

Ikegami and one of the authors (KK) have studied a population dynamics model with many types of hosts and parasites, which are subjected to mutations. In a weak coupling regime, a one-to-one relationship between a host and a parasite holds, while dynamic many-to-many relationships between pairs of hosts and parasites emerge in a strong coupling regime, together with the maintenance of a high mutation rate. We note that the resulting ecology is dynamically stable, sustained by a high-dimensional chaotic state, in contrast with the strong instability in a low-dimensional chaotic population dynamics. Some theory for dynamic many-to-many correspondences is required to allow for the diversity in an ecosystem.

In the present proceedings, **Yomo** presents the dynamic clustering of E-coli, bacteria. Even if these bacteria have identical DNA, they dynamically differentiate. The one-to-one correspondence between a genotype and a phenotype is invalid here. A novel mechanism for the differen-

tiation of cells is proposed, based on the idea of dynamical clustering (see **Kaneko and Yomo**).

Let us recall the history of Japanese literature. About three hundred years ago, it was popular to have ceremonies during which Haiku's (short poems) were recited. The ceremony staged poets who made poems in succession, following the previous poem by somebody else. A poet "interprets" the previous poem by him (her)-self. This interpretation, of course, may be different from the original poet's. Thus mis-interpretation is enhanced successively, but as a whole the sequence of poems forms some art more aesthetic than that created by a single poet. This process consists of the dynamic amplification of small deviations. "Collective" art at a higher level emerges as an ensemble of poems. One might think that this process is just a kind of bottom-up approach to collective art. This is not necessarily true. To address this problem, let us re-examine the top-down and bottom-up approaches.

3. Top-down, bottom-up, and emergence

There have been long debates between the top-down and bottom-up proponents in artificial intelligence and neural networks. In both approaches, it is assumed implicitly that the top level is represented by a few degrees of freedom, while the bottom level may involve a huge number of degrees of freedom. In the bottom-up approach some kind of "order parameter" constructed from the lower level is viewed as a representation at a higher level, related with some macroscopic behavior. In the top-down approach only a few instructions are sent as messages to lower-level elements.

As a natural compromise between these approaches, the inclusion of a weak feedback between the top/bottom levels has been proposed. An example is given by a simulation of ants with pheromone [15]. In the simulation, an ant emits pheromone when it has food, while other ants are attracted by pheromone through their mo-

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[1] This does not mean that such ecosystems are dominated by (few) keystone species. A role of a keystone species implicitly emerges within an ecological network, through the amplification of tiny causes as illustrated in the Japanese proverb.

tion. A collective field of pheromone is formed by the ants' motion. Since the dynamics of a lower-level unit (an ant's motion) is governed by the higher-level dynamics (collective field of pheromone), this scheme is analogous with the Prigogine's dissipative structure [5] or Haken's slaving principle [4].

In these approaches the relationships between elements are fixed. Although it may be possible to introduce nontrivial dynamics (e.g., in the ants' motion or in the field of pheromone), the behaviors of each element are passive and totally susceptible to a higher-level.

Most papers in the present proceedings adopt a different approach in the following senses. First, the top level is not necessarily represented by a few degrees of freedom; second, the relationships between elements at a lower level can often change dynamically. At first glance, the first point may just look like a complication. This is not necessarily so. Even in the midst of highly disorganized states, ordered motion governed by a few degrees of freedom often emerges, which, however, does not last for ever due to the second point (the dynamic change of relationships). Again, high-dimensional motion comes back, until another structure emerges. This mechanism, called *chaotic itinerancy* [17,23,13,24], can replace the views of the top-down and the bottom-up approaches. In a network of chaotic elements, for example, the order at the top level is destroyed by a chaotic revolt against the slaving principle [17], in contrast with passive elements in traditional approaches. In the dynamic neural network model by one of the authors (IT), the chaotic itinerancy leads to a spontaneous recall of memories [13]. A similar dynamical behavior is also observed in real brain activities. In the present proceedings, this topic is studied in the papers by **Tsuda, Aertsen, Dinse, Freeman**, and **Nozawa**.

In the population dynamics model mentioned in Section 2 [18], the higher-level corresponds to the collective dynamics for survival as an ensemble of many types. This higher level emerges

from the bottom level, but it is not necessarily represented by a few degrees of freedom. An ecosystem like **Ray**'s TIERRA, with species of programs, is not necessarily represented by a few sets of features. Species with different properties appear successively in a specific condition. In the paper by **Palmer**, a stock market is formed as a higher level.

The term "emergence" is often used as spontaneous appearance of an upper level description without explicit instruction for it [19]. If the upper level is represented by a few order parameters, the term "emergence", in this case, is just a rephrase of a dissipative structure in nonequilibrium statistical mechanics, or a collective behavior in equilibrium phase transitions. When the upper level is not represented by a few degrees of freedom, however, the emergence is no longer trivial.

The term emergence is often used when the behavior found in a computer experiment is not written in a model explicitly as an algorithm. In order to be an "emergent" behavior, an explanation of it from the implemented program should require at least as much as the information as the direct computation. Such "wishes" for the construction of emergence may not be rigorously accomplished as long as one uses a finite-state machine (e.g., digital computer) for a finite time interval, since the simulated behavior is obtained by a completely controlled program, with a finite amount of information. As long as all the information is finite, it is difficult to define a behavior "unexpected" from the implementation. Thus many people have tried to avoid this term in the workshop, although we tacitly feel that "emergence" is necessary for the understanding of the dynamics in brains and in biological evolution.

There can be two possibilities to remedy the above impossibility of emergence. One is the assumption of the use of infinite cells or tapes and/or a possible use of infinite time step computations. In connection with the undecidability of the halting problem, it may be possible to

have emergence by taking an infinite-step limit.

The other is the introduction of uncontrollability up to an infinite precision. Let us recall the Japanese proverb; given the strong wind, the outcome that the tub is sold well is rather unexpected. To explain this process, we have to follow each step of the reasoning, which itself is easily affected by a small error. The outcome, in this case, can be emergent behavior. By the introduction of an analog computer with chaotic dynamics and/or error in it, one may thus expect the occurrence of emergence. We note that notions of computability in a real-number machine, discussed by [25,26] in connection with chaos, may be essential to explore this possibility.

Another way for the introduction of uncontrollability may be the introduction of a quantum mechanical computer, as is discussed by **Conrad**.

Even in our digital computer of finite resources, there can be some hope. Chaos, for example, cannot be simulated rigorously by any digital machine: As long as a state in the machine is finite, the dynamics becomes periodic (Poincaré recurrence), finally. Still, we can grasp the features of chaos (by taking the limit of infinitely many states) in a digital computer. In a similar manner, emergence may be defined by taking the limit of infinitely many states from our digital machine. To understand the nature of this limit, we need to make more efforts to construct a model with some kind of emergence, and also some mathematical studies on the relationships between digital and other computers.

4. From a descriptive to constructive approach of nature

Structural stability [27] had been presented and recognized as a necessary condition of a real model, before the significance of chaos was appreciated. On the other hand, for a system with structural instability, a slight change of the model may lead to behaviors with different characteristics from real solution's. This is why such a system is believed not to be a good model for nature. However, structural instability can widely be seen in a nonlinear system including chaos. For example, chaos in the logistic map $x' = ax(1 - x)$ cannot exist in an open interval in the parameter space a. This means that a map with some fixed parameter has no topological equivalence in any neighborhood of that map, thus implying structural instability.

Chaos may have another transcendental nature. In some cases with non-uniform hyperbolicity, chaos may lack the pseudo-orbit-tracing property [28][2]. If so, this means that individual orbits in chaos cannot be traced by experiments or by numerical simulations. It is still questionable if a whole attractor, i.e., a strange attractor itself can be traced in experiments. At least one counter-example exists against the assertion that a strange attractor itself is traced. In some chaotic systems such as the Belousov–Zhabotinsky reaction map, chaos looses characteristics such as topological and measure-theoretic quantities, affected by noise, and consequently order that does not exist in any neighborhood of the original system appears [30]. A drastic change appears in the case of somewhat large noise, but the calculation of the Kolmogorov–Sinai entropy implies the creation of different chaotic systems even in case of infinitely small perturbations. This so called noise-induced order [30] has been interpreted in terms of an observational mismatch between the system's inherent observation window, i.e. Markov partition, and the external observation window forced by noise.

A complete description of chaos needs an infinite amount of information. By a slight change of coding, in the case of a chaotic system, the de-

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[2] For an example in a nonhyperbolic system, see e.g. [29]. So far we do not know a clear mathematical statement on the breakdown of the pseudo-orbit tracing property. See [30] for a possible example found heuristically.

scription of the system as, for instance, a finite automaton can change drastically.

Crutchfield has dealt with chaos as a class of various levels of finite automata. The input information is hierarchically classified as a language accepted by a machine that is constituted of chaos with a finite observation window and observed symbol sequences. Then, chaos appears as a kind of a finite automaton according to the respective observation window. One of the authors (KK) also discussed the dynamics of a coding scheme in a network of chaotic elements, where the coding tree of observed symbol sequence changes forever [17]. Any difference in the observation precision leads to a crucial change of the dynamics [20].

Thus chaos manifests itself in various forms, depending sensitively on its description. By these observations, one of the authors (IT) has introduced the term "descriptive instability", although further studies are necessary for its mathematical definition.

Noise-induced order, a coding tree in the network of chaotic elements, and Crutchfield's ϵ - machine have introduced novel viewpoints with regard to the 'observation' or 'description' in complex systems. Thereby, one may notice the need of a more extended concept than structural (in)stability in order to capture legitimately all the features of complex systems. Here, what we need is not a concept representing the system, but a concept about an 'observer' describing the system. Noise-induced order means that a change of the description of chaos brings about a distinct phase of chaos due to the "descriptive instability" of chaos.

The notion of "descriptive instability" raises a question about the validity of a descriptive way of modeling. Since a one-to-one correspondence is not possible for each elementary process, a descriptive approach that always accompanies analysis is not always relevant for the understanding of a complex system.

In physics, we are used to adopt a descriptive approach; for example, an equation at a macro-

scopic level (like the Navier–Stokes equation) is approximately derived from a microscopic level (like the Newtonian equation of many particles), and then numerically simulated. Conventionally, a model equation in physics is believed to have a one-to-one correspondence with the phenomenon concerned.

Studies on chaos, however, may lead one to question this traditional picture of nature. Let us take the example of chaos in fluid dynamics. If one carries out a splendid numerical simulation on sets of equations with the velocity and temperature fields (e.g., Navier–Stokes equation with buoyancy and heat), one possibly can get the same oscillatory behavior of rolls as in experiments. Does this success give any better intuition on the origin of this strange oscillation than that provided by a simple chaotic system? The authors think that the answer is "No" for most scientists who know about chaos. One of the most important lessons from chaos lies in that it has opened the road to a qualitative dynamical viewpoint. Low-dimensional chaos can provide a universal mechanism underlying the onset of turbulence.

By developing further the viewpoint of chaos, the importance of a constructive, rather than a descriptive, approach has been pointed out. An example of such a constructive approach is the coupled map lattice [16], proposed by one of the authors (KK) for the studies of spatiotemporal chaos, pattern dynamics, and so on. The model, constructed by combining some basic procedures (such as local chaos, diffusion, flow, ⋯), cannot be derived from a first-principle equation like in conventional physics, but it still has a strong predictive power for novel phenomenology classes in complex dynamical systems.

A model cannot be exactly the same as nature herself anyway. By "descriptive instability" there may not be a well-defined quantitative "distance" between a model and nature. Thus a descriptive model based on microscopic knowledge is not necessarily quantitatively very close

to the phenomena under consideration. Even if a descriptive model happens to be quantitatively close to nature, in a complex system, it is in principle intractable to check detailed correspondence between the model and nature, numerically or experimentally. Furthemore we often do not need the detailed information of nature which is sensitive to the details of the models. Rather, we are more interested in universal aspects robust against changes of the model. In other words, we go up to a higher-level description which focuses on structurally stable aspects. Thus a qualitatively correct model which forms some universality class [3] is strongly required, for which the constructive approach is often more powerful.

Through the constructive approach, one tries to understand how such phenomenology is legitimated, how large the universality class to be described by phenomenology is, and what the essence of the phenomena is. Only through this approach we can see why some type of complex behavior is common in nature, irrespective of the details, and then we can predict what class of systems leads to such behavior.

5. Why artificial reality

The constructive approach in the last section implies the necessity of the construction of a model with artificial reality. The behavior of a model is not easily derived analytically in complex systems. One needs computers as a heuristic tool, as a hypothesis generator, rather than as a descriptor. The activities often called "artificial life" belong to this class of modeling.

Such modeling is especially necessary when one deals with historical phenomena, like evolution, since it is rather difficult to understand one historical path, without knowing other could-be

paths. Construction of a model with artificial reality provides an alternative approach when the traditional one faces difficulties. In the present proceedings, papers by **Ray, Hogeweg, Lindgren, Suzuki, Ikegami,** and **Hübler** present successful examples, as well as the report by *Fontana* in the workshop [21]. **Palmer,** and *Yasutomi* [22] have shown the power of the artificial reality approach in economics.

Frequent criticism raised to the artificial modeling is the lack of quantitative predictions. In natural science, it is often presumed to be ideal to predict quantitative results obtained in quantitatively specified experimental conditions. Such a precise quantitative prediction is not available in artificial modeling. Still, the phenomena observed in the artificial reality can provide a metaphor for what occurs (has occurred) in nature and in human society. Furthermore we can understand the essence of "real" phenomena through the artificial world, which makes qualitative prediction possible in a much broader sense.

In the present volume **Li** discusses an expansion-modification system, a kind of cellular automata with a growing number of cells. The long-range correlation found in this "artificial model" made him and one of the authors (KK) to expect the existence of long-range correlations in real DNA sequences, which was later confirmed. Another example demonstrating a possible connection between artificial and "real" biology can be found in the paper by **Kaneko and Yomo** in this volume.

The significance of such "artificial science" was first pointed out by Simon, in the context of engineering science [31]. The present constructive approach to complex systems also has some applications to engineering problems, mainly in the area of information processing. In such a case, one constructs a system by combining procedures. Here we should note that the combination often leads to some (emergent) performance un-expected from the sum of procedures. We have some examples in these proceedings:

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[3] Here we use the term "universality class" as a qualitative class, thus, in a broader sense than adopted in statsitical mechanics.

Nozawa has shown that a combination of a network of chaotic elements and a neural network of Hopfield's type brings about remarkable efficiency in optimization problems. Kitano has demonstrated that the combination of a genetic algorithm and an L-system for developmental processes provides high efficiency in learning. Taga gives a beautiful application of the synchronization to bipedal motion, while Kopecz shows some emergent performance in robots whose motion is controlled by artificial neural nets, so that they successfully avoid obstacles.

6. Methodological problems in complex systems

As we have discussed so far, the behavior in an artificial world cannot be represented by reduced sets of degrees of freedom while exactly corresponding to the complex world. We need novel methodologies to understand the complex behaviors emerging in artificial models. So far we do not have established methods such as those in (equilibrium) statistical mechanics. We discuss briefly some possibilities.

(a) Multiple viewpoints.

It is often required to describe the observation from many points of view. For example, the understanding of pattern dynamics in spatiotemporal chaos requires both the views from real space and phase space. Integration of dynamical systems theory and computation theory is relevant to the understanding of complexity in chaos or cellular automata, as shown by Crutchfield and Mitchell. For game dynamics, we need viewpoints both from an algorithmic level of strategy, and dynamical systems theory, as is seen in the studies of Lindgren, Ikegami, Hübler, and Suzuki.

(b) Mathematical anatomy in a high-dimensional phase space.

In low-dimensional dynamical systems, "anatomical" methods of geometric structures in phase space have been developed. In complex systems, it is required to extend such anatomical studies to high-dimensional cases.

In a system with a static complexity such as the spin glass problem, the anatomy of the (energy/fitness) landscape has clarified its ruggedness [6], while, for a dynamic case, the studies are more difficult, (since "anatomy" itself is a static tool), although some pioneering approaches have recently been proposed [32–34] in Hamiltonian dynamical systems with many degrees of freedom (e.g., in molecular dynamics). The anatomical studies exploring the high-dimensional phase space will be important in systems with evolution and adaptability.

(c) Naturalists' viewpoints.

In complex systems, one possible direction is to make a collection of complex behaviors, list them up and then classify them. This approach, borrowed from natural history, has been adopted in complex systems. The classification of the behaviors of cellular automata by Wolfram [35], although not complete, can belong to this approach. Physicists' preference to make a "phase diagram" is the simplest version of such classification. In complex systems, we have to face more complicated classifications. The naive use of a phase diagram may not be powerful, there. Indeed, in cellular automata, the lack of a continuous parameter makes it difficult to construct a phase diagram following the classification (see also the paper by Mitchell in the present volume). If a system's dimension is very high, construction of a phase diagram is not practical, where one has to resort to more heuristic approaches. In such cases a naturalist's approach may be useful.

The three approaches mentioned above are not necessarily sufficient for understanding all complex systems. These are apparently the approaches from without[4], which is usual and common to conventional sciences. A constructivistic approach is different. A constructivist tries to make an uncontrollable world inside a

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[4] Here we use the word "without" as the antonym of "within".

computer which is controllable from outside, to make the world be functional. A decisive point for the success of the modeling lies in the construction of an internal mechanism. Therefore, a constructivist's approach is inevitably an approach from within. We discuss this point in the next section.

7. Internal observer

In a system with artificial reality, one has to construct an internal observer; otherwise a system can never be intelligent. In molecular biological systems, among others, Conrad [37], Rössler [36], Matsuno [38], and one of the authors (IT) [39] have pointed out the significance of an internal observer which reacts with high efficacy. In brain modeling also, it has been pointed out that the introduction of an internal viewpoint would be essential to understand 'the brain understanding itself'. If chaos works in many phases of brain activities, chaos can be a candidate of such an internal observer. This viewpoint has been called "chaotic hermeneutics" by one of the authors (IT) [39].

The significance of internal viewpoints for the "understanding" of systems was first proposed by Gödel [40] in constructing a theory to involve a description, from without, of formal system into again the formal system, thus a description from within. By this constructive approach, Gödel succeeded to prove that there exists a theorem which is true, but unprovable only by using theorems of the system. Hereby, the complexity of formal systems was elucidated.

Rössler has proposed a new scientific paradigm, that is, endophysics, generalizing the internal viewpoint in a formal system to that in physics, chemistry and biology. In endo-world, it could be that the observation *from within* differs from, even contradicts to the observation *from without*. Only the latter observation has been explicitly performed in conventional science. In complex systems, however, as has been discussed

here, the observation and description from without are apparently insufficient. Hence, one may well have to take the endo-viewpoint for a sufficient understanding of complex systems. **Rössler** has constructed chaotic models to recognize the world from within in these proceedings.

Through the approaches so far we are trying to make a reconstruction of the story for complex systems. This reconstruction is not necessarily unique. Since the stories can include many variables and parameters, it is not possible to conclude that only one of them provides the best model. Thus it could be said that Ockham's razor has lost its edge for complex systems we face now. **Rössler's** talk in the workshop seemed to absolve Ockham's razor.

References

[1] All the papers in the present volume are cited with the speaker's name in the workshop, which agree with the first author of the names in the present volume, except three cases: Yomo's presentation is given by the paper of E. Ko, T. Yomo and I. Urabe. An additional paper by Kaneko and Yomo is inserted, since it is related with the experimental paper by Ko et al. Michael Conrad's presentation is given by the paper of J-C. Chen and M. Conrad, while Hübler's presentation is given by the paper by A. Pierre, A. Hübler and D. Pines. Dinse, who was invited but could not attend, is so kind to submit his could-be presentation in the volume.

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